Instrument Landing System Lateral Beam Guidance System Based on Sliding Mode Control Technique

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Resumen: *En este artículo se muestra un entorno basado en Matlab-Simulink para el análisis de un sistema de aterrizaje de un avión que puede ser utilizado para el aterrizaje automatizado o asistido. Se describe la construcción de un modelo matemático que posteriormente es simulado para validar su funcionamiento. Finalmente, el controlador por modos deslizantes es implementado y se analiza el comportamiento del sistema controlado incluso tomando en cuenta perturbaciones.*

Palabras clave: *SlidingMode Control, Control No-lineal, Teoría de Control, Aeronaves.*

Abstract: *This paper shows an environment based on Matlab-Simulink to analyze a landing system of an aircraft which can be used for automated or assisted landing. Firstly, it is described the development of a mathematical model which is simulated to validate its performance. Finally, the sliding mode controller is implemented, and the behavior of the controlled system is analyzed even taking into account disturbances.*

Keywords: *Sliding Mode Control, Non-linear control, Control Theory, Aircrafts.*

1. INTRODUCTION

The ground-based elements of an Instrument Landing System (ILS) are composed of a localizer transmitter, a glideslope transmitter and marker beacons. These provide the azimuth, vertical and distance signals respectively (Fig. 1). [1]

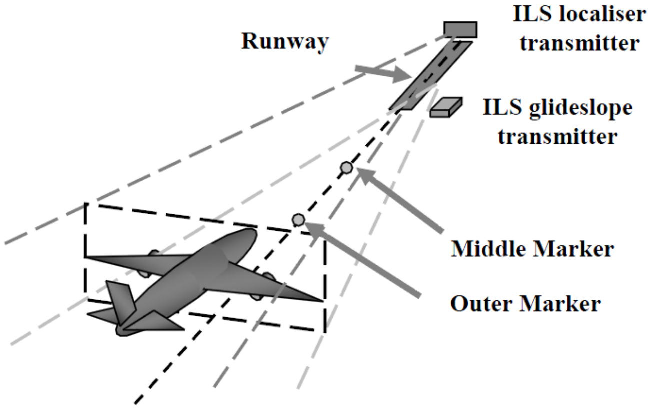

Figure1. ILS Diagram

On the aircraft isinstalled a localizer antenna, a glideslope antenna, an ILS receiver unit and a marker beacon antenna and receiver. The position of the aircraft relative to the localizer and glideslope is displayed on an indicator in the cockpit and is used to land safely. [1]

The Localizer System

"The localizer transmitter is located at the end of the runway which the aircraft is approaching. It transmits on a given frequency in the band 108 MHz to 112MHz. The signals radiate to the left and right of the centre line of the runway as shown in Fig. 2. The signalto the left is modulated by a 90 Hz component while the corresponding frequency for the signal on the right is 150 Hz. The two signals overlap in the middle"[1]. If there is predominance either of 90Hz or 150 Hz means that the aircraft out of the centre and it must move to the right or the left until reach the centre of the line.

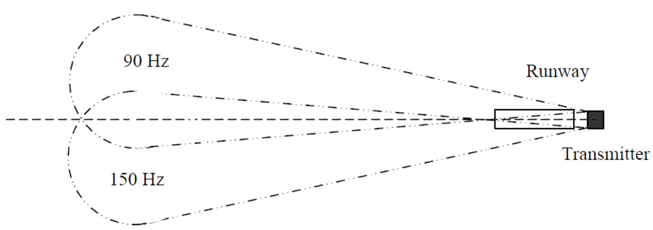

Figure2.The pattern of the localizer

The Glideslope System

The glideslope transmitter is an antenna located near the point of touchdown on the runway and transmits on a given frequency in the range 329.3 MHz and 335.0 MHz. The radiated signal pattern is similar to that of the localizer but provides vertical guidance relative to a descent path (Fig. 3) [2]

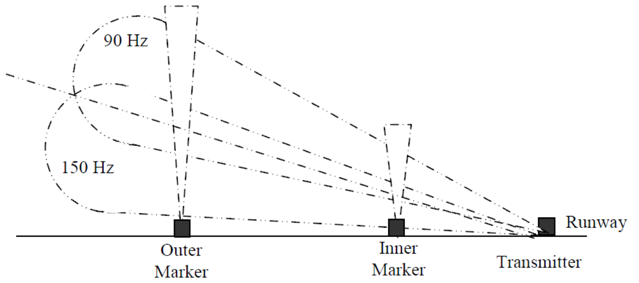

Figure3. The Glideslope system

Marker Beacons

Marker beacons give 75 MHz signals beamed vertically. These markers display the distance of the aircraft to the runway, so that the speed during the descent can be controlled by the pilot.

2. MODELLING THE SYSTEM

The system of the lateral beam guidance is shown in Fig. 4.

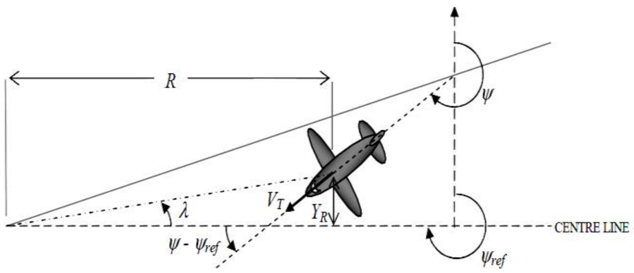

Figure4.System of Lateral Beam Guidance

The system which is going to be used in Matlab is presented in Fig. 5.

The system contains three feedback loops. The outermost loop involves the feedback of the heading angle. The other two loops provide feedback signals needed to ensure that the whole system has a reasonable dynamic response.

The reference is the heading angle, it could be provided by the pilot or as an electronic signal from other system in the aircraft [1].

The transfer function that relates the roll rate (p) to the aileron deflection angle delta (δ) is stated in the equation (1).

$$\frac{p(s)}{\delta(s)} = \frac{Ka}{sTa+1} \quad (1)$$

Where Ka is a gain factor and Ta is a time constant associated with this simplified model. Similarly, the relation between δ and the roll angle PHI Φ is the equation (2).

$$\frac{\emptyset(s)}{\delta(s)} = \frac{Ka}{s(sTa+1)} \quad (2)$$

The aileron servo is represented by the following relationship:

$$\frac{1}{sT+1}(3)$$

The three transfer functions described above can be converted into a state space form as in the equations (4), (5) and (6).

$$\frac{dp}{dt} = -\frac{1}{Ta}p + \frac{Ka}{Ta}\delta(4)$$

$$\frac{d\emptyset}{dt} = p(5)$$

$$\frac{d\delta}{dt} = -\frac{1}{T}\delta + \frac{1}{T}e_s \quad (6)$$

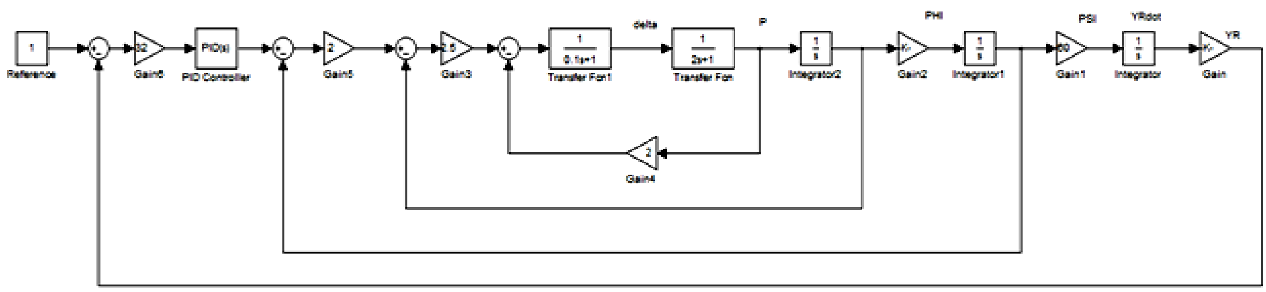

Figure 5.Block diagram of an aircraft directional-control system

The roll angle and the heading angle are related by the following equation:

$$\frac{d\psi}{dt} = \frac{g}{Vt}\emptyset (7)$$

where g is the gravitational constant. The structure of the feedback system is defined by the following equations:

$$e_s = K_v e_v - K_r p (8)$$

$$e_v = K_d e_d - \emptyset (9)$$

$$e_d = \psi c - \psi (10)$$

Fig. 5 shows that the Lateral Beam Guidance has a reference which is compared with the output. This comparison is made by a PI controller. The form of this PI controller is the following:

$$C = G_c\left(1 + \frac{k_i}{s}\right)\left(\lambda_{ref} - \boldsymbol{\lambda}\right)(11)$$

In order to estimate the angular error between the aircraft and the centre line of the runway, the equation (12) is going to be considered.

$$\sin(\boldsymbol{\lambda}) = \frac{\boldsymbol{Y_R}}{\boldsymbol{R}}(12)$$

For small angle approximation, the equation (12) becomes equation (13)

$$\lambda = \frac{Y_R}{R} \quad (13)$$

From the equations described above is possible to obtain the state space model for this system. The first step is to define the state vector, which is the following:

$$\begin{bmatrix} \lambda \\ Y_R \\ \psi \\ \emptyset \\ p \\ \delta \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}$$

Once defined the vector state, the second step is to take the derivative of every state of the vector.

$$\begin{bmatrix} \dot{\lambda} \\ \dot{Y}_R \\ \dot{\psi} \\ \dot{\emptyset} \\ \dot{p} \\ \dot{\delta} \end{bmatrix} = \begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \\ \dot{x_4} \\ \dot{x_5} \\ \dot{x_6} \end{bmatrix}$$

Finally, equations (1) to (13) are combined and the state variables are substituted and reduced into the derivative equations.

The first state **λ**is obtained from the derivative of the equation (13)

$$\dot{\lambda} = \frac{\dot{Y_R}}{R}$$

where $\dot{Y_R} = Vt\psi$, thus:

$$\dot{\lambda} = \frac{V_t}{R}\psi$$

or

$$\dot{x_1} = \frac{V_t}{R}x_3 \quad (14)$$

Since

$$\dot{Y_R} = Vt\psi$$

Then

$$\dot{x_2} = Vtx_3 (15)$$

From equation (7)

$$\dot{x_3} = \frac{g}{Vt}x_4 (16)$$

$$\dot{\emptyset} = p$$

$$\dot{x_4} = x_5 (17)$$

From equation (4)

$$\dot{x_5} = -\frac{x_5}{Ta} - \frac{K_a}{T_a} \quad (18)$$

From (6), (8), (9) and (10)

$$\dot{x_6} = -\frac{x_6}{\tau} + \frac{KvKd}{\tau}x_3 - \frac{k_v}{\tau}x_4 - \frac{k_r}{\tau}x_5 \quad (19)$$

Equations (14) to (19) are the state space model of the system that is going to be used in Matlab.

3. SIMULATION OF THE SYSTEM

Fig. 6 to 11 show the behavior of the system when the reference is 15 degrees. **λ**does not have equilibrium point, it grows without control as it is shown in the Fig. 6. Fig.7 shows that the altitude of the plane is not set to 6000m, which is the initial condition for this. Fig.8 describes the heading angle of the plane. This angle should be zero. It is seen in the figure that there is a jump from zero to 15 degrees and slowly it comes back to zero after 450 seconds. Fig.11 can be seen a deflection in the aileron during 10 seconds. This leads to a change in the roll rate and in the roll angle. As it is shown in Fig.9 the roll angle has a transient during 10 seconds reaching a maximum value of 4.4 degrees after that the plane tries to be stable at zero degrees again. In general the behavior of the system is unstable even if the system does not present any external disturbance. The next step is to include a controller to make the system stable.

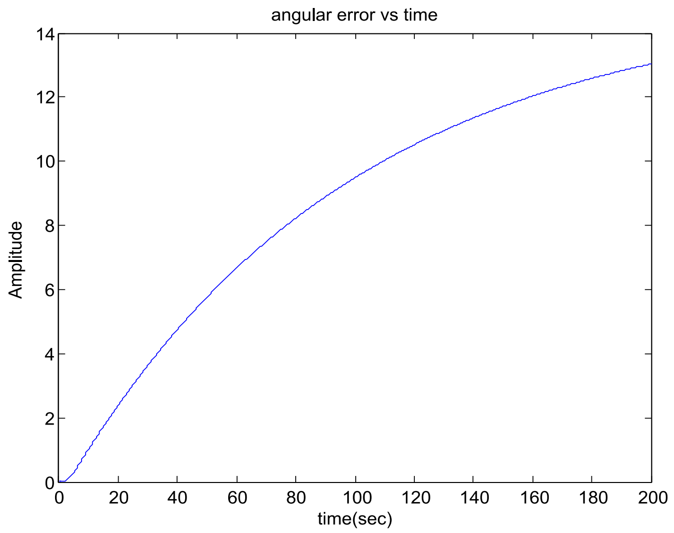

Figure6. The error angle$\boldsymbol{\lambda}$ when the reference is 15 degrees without controller

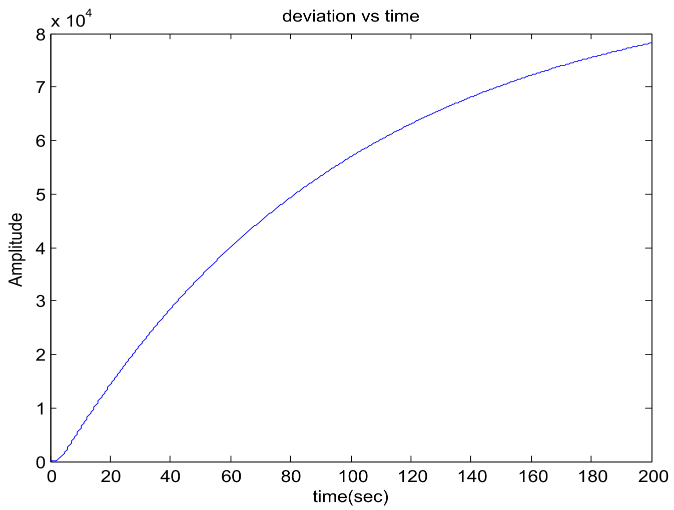

Figure7. The deviation YR when the reference is 15 degrees without controller

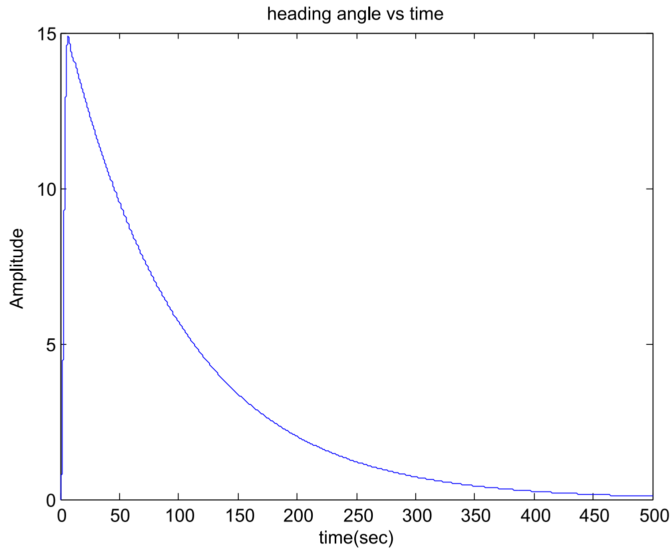

Figure8. The heading angle ψ when the reference is 15 degrees without controller

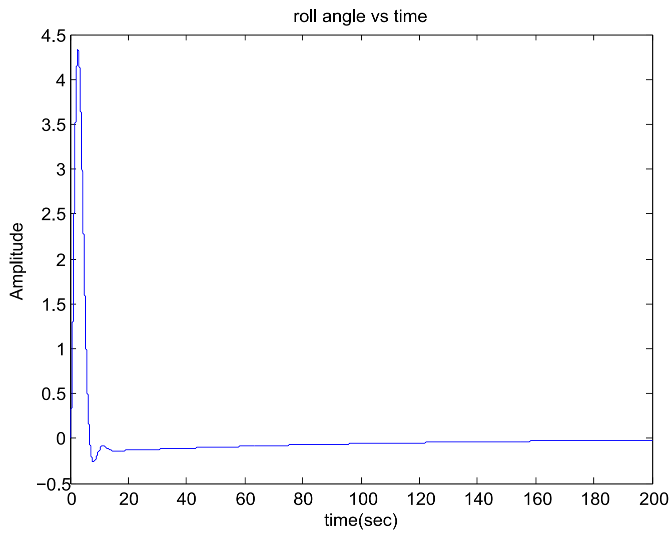

Figure 9. Roll angle ψ when the reference is 15 degrees without controller

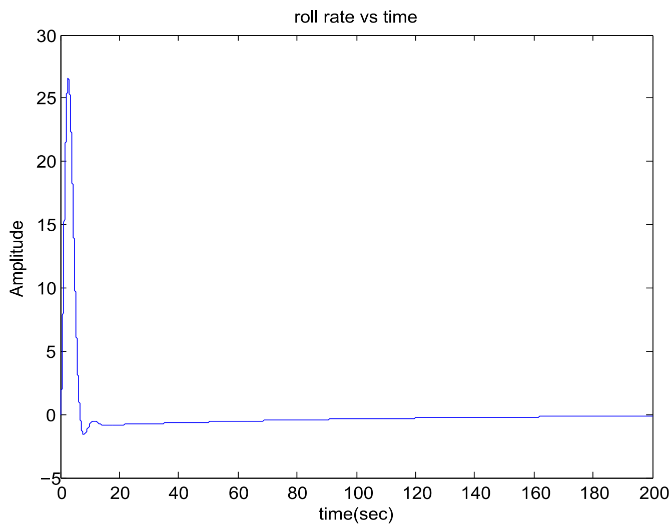

Figure10. Roll rate p when the reference is 15 degrees without controller

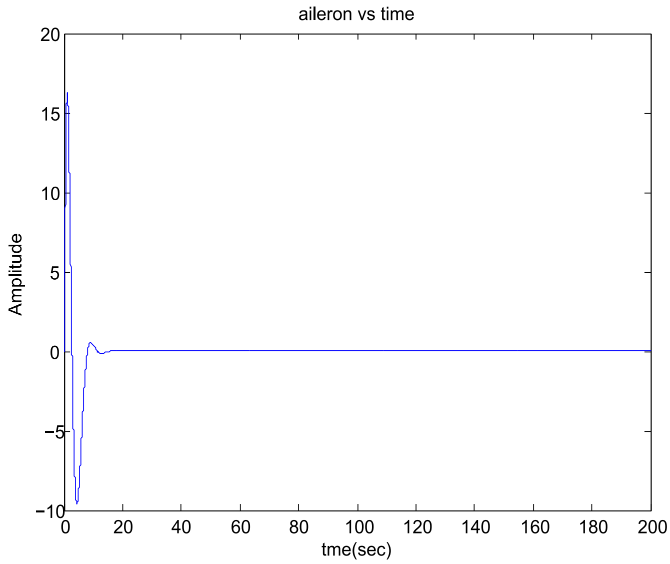

Figure11. aileron deflection δ when the reference is 15 degrees without controller

4. LONGITUDINAL DYNAMICS

In principle, the system becomes stable by using a PI controller when the reference is a small value. Now, the longitudinal dynamics of the aircraft is going to be considered. See in table 1how the range of an aircraft on ILS approach changes with time:

Table1. Variation in time

t(s)	R(m)
0	6000
10	4675
20	3640
30	2835
40	2210
50	1720
60	1340
70	1045
80	810
90	630
100	495

The linear interpolation routine is going to be implemented to determine the range values that fall between these data points.

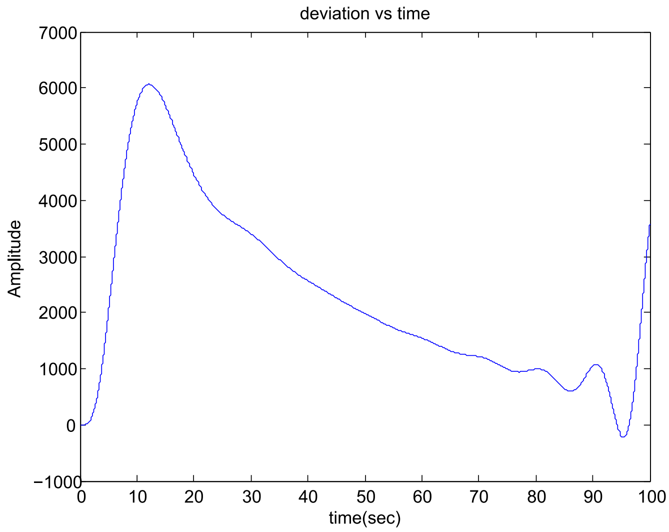

Figure12. Deviation YR in time

Fig. 12 shows the deviation of the system, when Gc = 16.63 and Ki = 0.0028 in the PI controller. After some tests, was found that the system behaves properly using these values of Gc and Ki. The initial position is 6000m. The beginning of the curve shows a jump from zero to 6000m because the input signal is a step function used only for simulation. For these values of coupler, the system tries to follow the trajectory in correct manner going down from 6000m, but at around 85 seconds, the system shows oscillations, which means that the system becomes unstable.

Looking at Fig. 4 and the equation (13),it is concluded that the system is more sensitive when the plane comes to O, because **λ** increases while R decreases. Fig. 12 clearly shows that the system is stable for large values of R, but it becomes unstable when R becomes smaller.

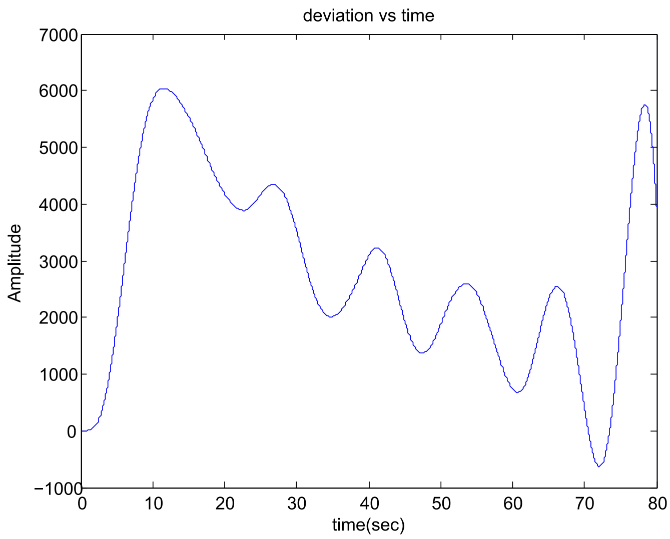

Figure13. Deviation in presence of disturbances.

According to Fig. 13, the system is unstable when external disturbances are present. It is clearly seen that during the trajectory the system presents high oscillations and at 70 seconds it loses completely the stability.

5. SLIDING MODE CONTROL

In order to reduce the effect of disturbances and to make the system more robust, a non linear control technique is going to be implemented, which is the sliding mode control technique.

The main idea to control a system by using sliding mode control is to make the states of the system converge to a sliding surface and make them stay on it, as it is shown in the Fig. 14. In this way, the dynamics of the system is defined by the equations that determine this surface.

Establishing these equations and making them act on the system makes possible to obtain the stabilization of the system, a precise set point following and the regulation of the variables.

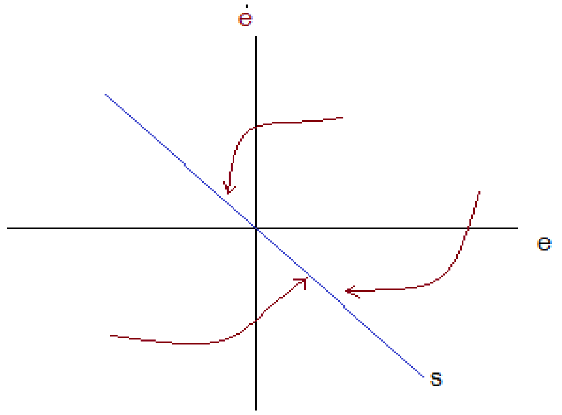

Figure14.Sliding Surface

The sliding surface, noted as "s" is defined by the equation (20)

$$s = \dot{e} + \alpha e \qquad (20)$$

whereα is a parameter design greater than zero.
In order to reach the main idea of the sliding mode control, the control law will be defined by the equation (21)

$$u = ueq + ucr. \quad (21)$$

The equation (21) shows that the control law has two elements which are ueq and ucr. ueq is the equivalent control and this is the part of the controller that maintains the state of the system restricted to the sliding surface. ucr is the correction control and this is the part that make the state to converge the sliding surface. Hence, the controller is going to have the structure of the equation (21).

The sliding surface is defined in the equation (20), and e is the error which is defined as the difference between the measured state and the desired state. For this system the angular error **λ** is going to be the variable that is wanted to be controlled, so the error for our system will be:

$$e = \lambda - \lambda d \qquad (22)$$

where**λ** is the angle measured and **λ**d is the desired angle.
Replacing (22) in (20), the sliding surface is the equation (23).

$$s = (\dot{\lambda} - \dot{\lambda d}) + \alpha(\lambda - \lambda d)(23)$$

The equation (24) is the law for the attractive surface, which is the derivative of (23).

$$\dot{s} = \left(\ddot{\lambda} - \ddot{\lambda d}\right) + \alpha\left(\dot{\lambda} - \dot{\lambda d}\right) \quad (24)$$

From (5), (7) and (13) the following relationship is obtained.

$$\ddot{\lambda} = \frac{Vt}{R} p \quad (25)$$

Replacing (25) in (24) the following relation is found.

$$\dot{s} = \left(\frac{Vt}{R} p - \ddot{\lambda d}\right) + \alpha(\dot{\lambda} - \dot{\lambda d}) \; (26)$$

In this case p = ueq+ucr. In order to calculate ueq $\dot{s}$ and ucr are going to be considered as zero; therefore, ueq is defined by the equation (27).

$$u_{eq} = -\frac{R}{Vt}\alpha\left(\dot{\lambda} - \dot{\lambda d}\right) + \frac{R}{Vt}\ddot{\lambda d} \qquad (27)$$

Once defined the sliding surface s, ucr is the relation stated in the equation (28).

$$u_{cr} = -sign(s).\eta \qquad (28)$$

where η is another design parameter and greater than zero.
The implementation of this gives the result plotted in Fig. 15.

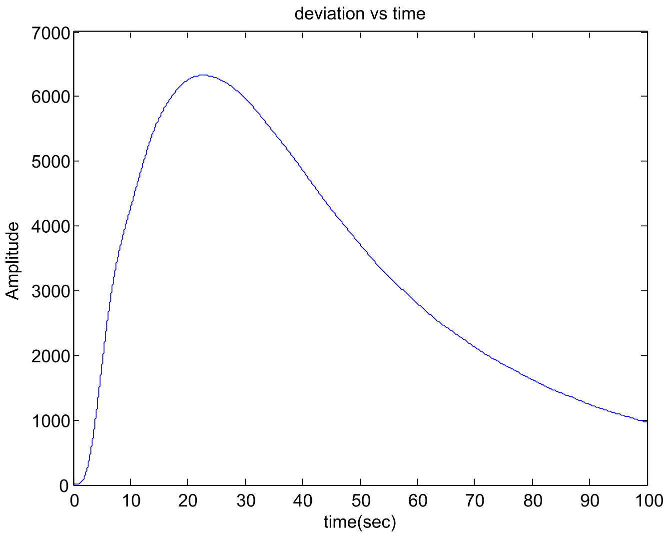

Figure15. Deviation using sliding mode control.

The constants α and η found were 0.5 and 20 respectively. With this, the system is stable even for low values of R, which was a problem by using a PI controller.
When there is a disturbance, the system responses in a better way, during the trajectory, it is possible to see oscillations, but this time they are very small as it is presented in Fig. 16.

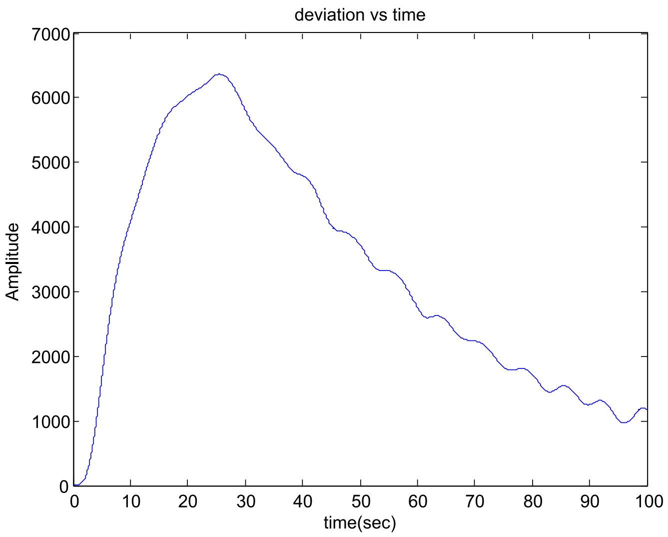

Figure 16. Deviation using sliding mode control and disturbances.

6. CONCLUSIONS

In order to make the system more stable, the sliding mode control technique was implemented. The results showed a stable system even for low altitudes and considering external disturbances. The sliding mode controller is precise because it was designed considering the non linearities of the plant. The sliding mode control gave better results. It is recommendable to try more non linear techniques like fuzzy or H infinity which would improve the stability and the robustness as well.

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