Parametric Research of Granular Flow in Silos: A Micro-Mechanical Approach

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Abstract: The study of granular materials stored in silos is traditionally conducted with postulates and definitions of the continuum mechanics. Specific interactions of the granular matter into the silo (e.g. contact forces or velocity) are not quantified in this theory. Considering this limitation, the purpose of this research is to study the granular flow of corn particles and their interactions during the silo discharge by means of micro-mechanical methodologies i.e., the discrete element method (DEM). DEM is a numerical technique that allows to model granular assemblies based on their mechanical, physical properties and interactions. In this study, assemblies constructed with representative particles of corn have been developed. Velocity profiles, stresses in the silo walls, force chains and deformations of the bulk are the generated outcomes after running the simulation cases. In conclusion, the repose angle of the stored material plays a starring role in the mechanical response of the granular matter in the silo. Wall stresses, force chains and deformations increased when the silo hopper is lower than the repose angle of the corn granular assembly (27°).

Keywords: Corn; discrete element method; granular flow in silos; repose angle; hopper angle

Estudio Paramétrico del Flujo Granular en Silos: Una Aproximación Micro-Mecánica

Resumen: El estudio del material granular almacenado en silos se lo ha realizado habitualmente con las formulaciones de la mecánica del medio continuo y los elementos finitos. Sin embargo, existen diversas limitaciones al cuantificar la interacción entre partículas y su comportamiento individual. Por lo tanto, se plantea la utilización del método del elemento discreto (DEM) para evitar las limitaciones intrínsecas de modelos continuos en el análisis del flujo de maíz (materia granular) durante los procesos de descarga en silos. El elemento discreto es una eficaz herramienta mecánico-computacional que permite modelar ensambles granulares al considerar sus propiedades físicas y mecánicas tanto al nivel individual como de conglomerado. En esta investigación, los ensambles diseñados son representaciones numéricas de granos de maíz almacenado en silos. Los resultados de las simulaciones se cuantifican en términos de perfiles de velocidad, cadenas de fuerza, esfuerzos en las paredes del silo, y deformaciones del conglomerado granular. Uno de los principales hallazgos de esta investigación es la importancia del ángulo de reposo del maíz en la descarga de silos ya que los esfuerzos, deformaciones y cadenas de fuerza varían dependiendo de este valor (27°).

Palabras clave: Maíz; método del elemento discreto; flujo granular en silos; ángulo de reposo; ángulo de la tolva

1. INTRODUCTION

Granular materials are assemblies constructed with particles whose behaviour differ at single and conglomerate level depending on the influencing environment (Goodman & Cowin, 1972; Savage, 1979). At static conditions the material can be considered as solid, but, whether the particle conglomerate moves, its behaviour is considered as fluid (Elaskar & Godoy, 2001). Productive processes in industry (e.g. pharmaceutical, mining or agriculture) handle granular materials as the main source of production (Hill, 2012). More than 50% of the raw materials around the world are constructed or distributed in granular bulks (Boac, 2010; Gustafsson, 2008). In addition, more than 10% of the global energy consumption is utilised whilst transporting and processing them (Ishkov, 2016). Particularly, storing the material for long time under adequate conditions (e.g. temperature and humidity) have led to design reservoirs entitled as ‘Silos’. Although these vessels have been largely

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studied, the complex interactions in the granular flow at micro and macro-scale have not been fully understood yet.
The contemporary design of silos applies formulations of the continuum mechanics to simplify the particle interactions in it (Wang et al., 2015). Wall pressure profiles, dynamic meshing (i.e. granular flow modelled as fluid) and flow patterns (i.e. funnel or mass flow (Mankoc et al., 2007; Rotter et al., 1998)) are typical assumptions implemented in continuum models (Brown, 2007; Wang et al., 2015). Furthermore, constitutive models (e.g. Drucker-Prager or Frictional contact) are used to model the elastoplastic interactions occurring within the silo (Elaskar & Godoy, 2001; Wieckowski, 2003). Stress and strain profiles in both the structural walls and assumed fluid are the outcomes of these models. This information is helpful when selecting the thickness and type of material to build the silo, however, realistic predictions or estimations of the granular flow during the silo discharge are not clearly described. For instance, jamming, which can be defined as a sudden stop of particle flows (Zuriguel et al., 2005), is a complex process to explain uniquely by finite formulation. Under these circumstances, the Discrete Element Method (DEM) appears as a feasible alternative to elucidate the complex interactions into the silo at micro scale (Cundall & Strack, 1979).

The robustness of the DEM compared with other formulations is its flexibility to consider each component of the granular assembly as a unique solid element that interacts with their surrounding neighbours (e.g. particle or walls) (Andrade et al., 2012; Cundall & Strack, 1979; O'Sullivan, 2011; Queteschiner & Kloss, 2010). Bulk properties (e.g. friction coefficient) or individual properties (e.g. density) are defined whilst modelling complex granular assemblies (Baars, 1995). DEM-based schemes analyse both collisions among particles and between particle and walls. The outcomes of DEM simulations are contact forces between the interacting elements, position and velocity of each particle (Cundall & Strack, 1979). Considering the benefits of DEM, investigations into further describing the granular flow in silos have been undertaken. Parafiniuk, Molenda and Horabik (Parafiniuk et al., 2013) analysed the granular flow in silos under two considerations dry and wet conditions. They found that their experimental results agreed with the numerical simulations when slow flow rates are modelled. Another interesting analysis to predict the relation between the flow rate and the silo discharge orifice was performed by Zhou et al. (Zhou et al., 2017). It was numerically predicted that both conditions interact in the volume fraction and particles velocity. In this research, 2D assemblies were applied to compare with 3D laboratory experiments. Other recent investigations limited their studies to analyse how the silo hopper or the bulk packing fractions alter the flow rate during the discharging process (Benyamine et al., 2017; Tan et al., 2016).

The purpose of this research was to develop micro-mechanical models to resemble the discharge of corn particles stored in silos to (1) identify the influence of the hopper geometry in the particle kinematics and (2) the geometry influence in the granular flow, particularly, in terms of the stresses-strains in the discharging region. The achievement of these objectives will increase our understanding of the complex interactions between the granular matter and the silo walls.

2. MATERIALS AND METHODS

2.1. Discrete Element Method

Granular assemblies based on the DEM will be elaborated to model the transient interaction of particles during the discharge of silos. This methodology utilises the postulates of the Newton´s Law to quantify grains interactions (O'Sullivan, 2011). The equation of motion (Equation 1) describes the interacting forces in granular arrangements:

$$m_p\overrightarrow{\ddot{u}_p} = \sum_{c=1}^{Nc,p} \vec{F}_{pc}^{con} + \sum_{j=1}^{Nc,p} \vec{F}_{pj}^{non-con} + \vec{F}_p^{f} + \vec{F}_p^{app} \qquad (1)$$

Where:
m_p: *mass of the particle*
$\overrightarrow{\ddot{u}_p}$: *acceleration of each particle*
$\vec{F}_{pc}^{con}$: *contact forces between particles and their neighbours*
$\vec{F}_{pj}^{non-con}$: *non-contact forces with the particles*
$\vec{F}_p^{f}$: *gravitational force*
$\vec{F}_p^{app}$: *applied forces onto a particle including the contact particle-wall*

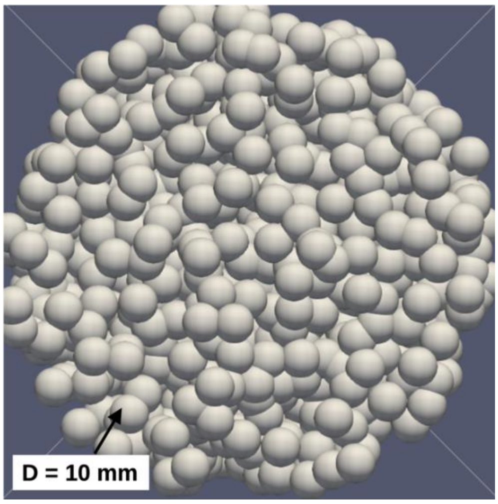
This mathematical expression considers both the individual qualities of the particles and their interacting states. In addition, the rotational and translational motions of each element are also considered. To assess the contact among particles ($\vec{F}_{pc}^{con}$), formulations that analyse the contribution of each element in the system were proposed (Cundall & Strack, 1979; O'Sullivan, 2011; Queteschiner & Kloss, 2010). The soft sphere model takes advantage of the Hooke's law and the Hertz contact model to characterise the overlap between interacting particles by means of the contact stiffness, Young's Modulus (E) and Poisson ratio (υ) (Babié, 1988; O'Sullivan, 2011; Queteschiner & Kloss, 2010). Moreover, the soft-sphere model used in this study depends on a rheological model composed of a linear spring-dampshot (Queteschiner & Kloss, 2010). The model includes a tangential spring to compute the tangential force during the relative tangential motion between particles. The tangential force is directly influenced by the Coulomb frictional limit that depends on the coefficient of friction between particles (μ_{corn}) or between the particle and contact surface (μ) (Queteschiner & Kloss, 2010). It is worth mentioning that the interaction occurs at short instants of time, based on this, the dispersed energy at each contact is assumed to be transmitted by Rayleigh-waves (TR) (Queteschiner & Kloss, 2010). This formulation (TR) considers the density (ρ), particle radius (R), Poisson's ratio, and shear modulus (G) of the granular media; shown in Equation (2. The solution of this equation (ranges around 0.1TR – 0.3TR) is used to estimate the time step (Δt) of the DEM problem case.

$$T_R = \pi R \frac{\left(\frac{\rho}{G}\right)^{\frac{1}{2}}}{(0.1632\nu + 0.8766)} \qquad (2)$$

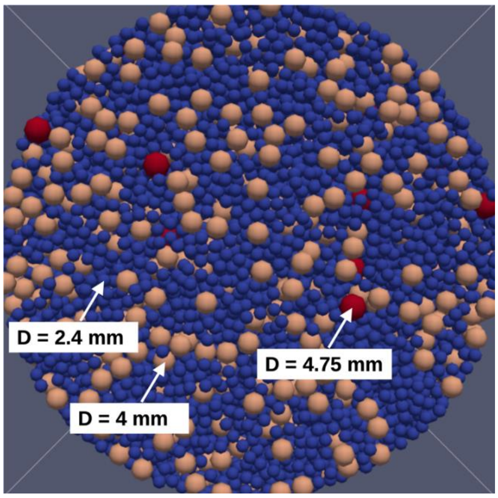
The previous information is feasible to handle in small conglomerates with a limited number of particles. However, real-life problems or even scale models group thousands of interacting elements increasing the number of contacts in the assembly. Because of this, the mathematical background which defines the DEM is implemented in numerical algorithms to manage relatively large granular assemblies (25,000 particles). In this research, the LIGGGHTS open-software is the computational tool utilised to model the silos' discharge.

2.2. Granular assemblies

Granular assemblies are sets of particles that resemble the real granulometric distribution of granular matter in a specific region. Based on this, two particle distribution alternatives which are monodisperse and polydisperse assemblies can be developed. Monodisperse models are constructed with particles of the same size; shown in Figure 1(a). On the other hand, granular elements of different sizes conform a polydisperse assembly, see Figure 1(b). This study developed polydisperse models that resemble corn stored in silos.



(a)



(b)

Figure 1. (a) monodisperse model constructed with iron ore pellets particles (diameter = 10 mm) and (b) polydisperse model built with corn

2.3. Particle and conglomerate properties

In this study, particles of corn were modelled because of the importance of this granular material in agriculture and productive processes. The definition of granular assemblies in DEM is accomplished on the basis of their individual and conglomerate properties. For corn, there are diverse studies investigating its mechanical properties at micro and macro-scale (Ileleji & Zhou, 2008; Moya et al., 2013). Each corn particle was modelled as sphere in order to reduce computational costs and increase the number of interacting elements in the assembly. The granular assemblies generated were polydisperse considering the granulometry of corn. The properties of the granular assembly and granulometry are shown in Table 1 and Table 2, respectively. To determine the repose angle of corn, we conducted an experimental test following the methodology used in a previous study by Perazzo et al. (2019), which investigated the repose angle of copper ore. The results revealed that the repose angle of corn was 27°.

Based on the experimental results and in line with published data (Moya et al., 2013), we have determined the friction coefficient between corn particles to be 0.51 ($\mu_{corn} = 0.51 = \tan(27°)$). Similarly, for the friction coefficient between corn and the concrete walls of the silo, the chosen value is 0.52, also in accordance with published data (Moya et al., 2013).

Table 1. Properties of the granular assembly (Boac, 2010; Fernández, 2010; Ileleji & Zhou, 2008; Lira & Pina, 2011; Moya et al., 2013)

	Particle level	
	Corn	**Concrete**
Young's Modulus (MPa)	298	50,000
Poisson's ratio	0.3	0.2
	Conglomerate level	
	Corn – Corn	**Corn –Concrete**
Restitution coefficient	0.205	0.700
Friction coefficient	0.510	0.520

Table 2. Corn granulometry

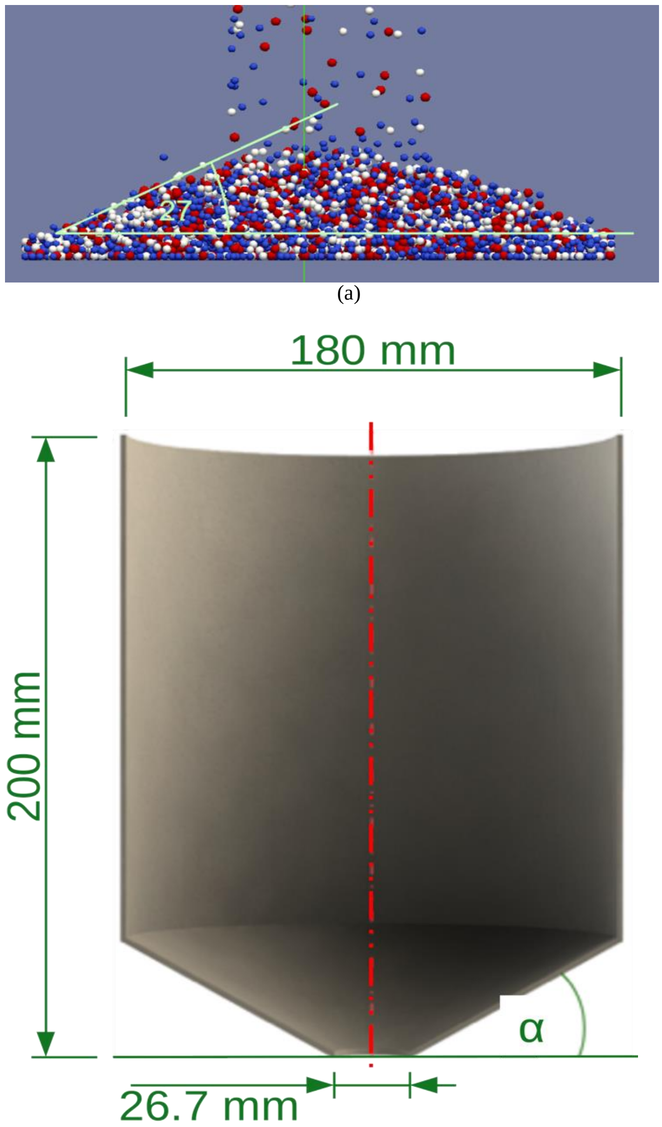
Sieve size	**Aperture (microns)**	**Retained weight (g)**	**% Retained**
1/2	12,700	0.00	0.00
3/8	9,500	11.60	2.32
5/16	7,940	186.35	37.27
4	4,750	301.25	60.25
8	2,360	0.80	0.16
Total		500.00	100.00

2.4. Characterisation of the silo

The silo walls were numerically characterised according to their mechano-physical properties (e.g. restitution coefficient particle and wall). In this research, silos constructed in concrete were modelled because they are widely used in industry for storing granular materials; particularly, corn. Table 1 summarises the relevant properties utilised to define silos of concrete. In geometrical terms, to identify the interaction of the granular discharge flow and the silo walls, several geometries with different hopper inclination were designed; shown in Figure 2(a). The baseline to construct the silo chute was the repose angle of the stored media (i.e. 27° for corn); see Figure 2(b).

2.5. Computational design

Considering the high computational costs that entails modelling complex arrangements by means of the Discrete Element Method, the size of the container and number of particles has been limited. 25,000 corn particles were stored in each study case and various silo geometries were designed to evaluate the granular flow in it. In order to guarantee convergence and stability in the numerical method, the timestep was assigned as 1x10^{-6} sec. This value was useful to define the total discharging time of the silo; approximately, 40 sec. The simulation cases were run in a serial computer with a multicore processor of 2.73 GHz. The simulation time varied between 70 to 100 h. Considering that the algorithm to generate the granular assemblies insert corn particles into the silo randomly, 3 tests per silo condition were implemented.



(a)

Figure 2. Characterization of the granular assembly (a) repose angle of corn and (b) geometry of the silo

2.6. Assessment of deformations in the granular assembly

Assessment of deformation in the granular corn assembly was performed in the boundaries of the system due to the inherent limitations in the soft sphere model that make challenging the computation of overlapping distances among contacting particles. Two mathematical approaches, which are the convex hull and Delaunay triangulation, were used to identify the boundaries (i.e. control volume) of the corn assembly stored in the silo. The convex hull identified the particles that delimited the control volume, and the Delaunay triangulation identified the contacting particles at the convex hull. Computation of the average deformation ($\bar{\varepsilon}$) consists of the vectorial product of the average displacements ($\boldsymbol{u}^n$) of the particles (i, j, k) of each triangle identified by the convex hull and Delaunay triangulation and the normal vector ($\boldsymbol{v}^n$) to the surface of each triangle (Avila & Andrade, 2012). This product is multiplied by the area (A^n) of each triangle and divided by the volume of the convex hull (Equation 3).

$$\bar{\varepsilon} = sym\left[\frac{1}{V}\sum_{n=1}^{N} \boldsymbol{u}^n \otimes \boldsymbol{v}^n \, A^n\right] \qquad (3)$$

After completing the evaluation of Equation (3), the average deformations were quantified in a tensor (Equation 4), which after evaluation of the first invariant (I_1) and the second invariant of the strain deviator tensor (J_2) allowed us to evaluate the volumetric changes (Equation 5), and angular deformation (Equation 7) of the corn assembly. The evaluation of J_2 entails the calculation of the second Invariant (I_2) by means of Equation 6.

$$\bar{\varepsilon} = \begin{pmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{12} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{13} & \varepsilon_{23} & \varepsilon_{33} \end{pmatrix} \qquad (4)$$

$$I_1 = \varepsilon_{ii} = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33} \qquad (5)$$

$$I_2 = \begin{vmatrix} \varepsilon_{11} & \varepsilon_{12} \\ \varepsilon_{13} & \varepsilon_{22} \end{vmatrix} + \begin{vmatrix} \varepsilon_{11} & \varepsilon_{13} \\ \varepsilon_{13} & \varepsilon_{33} \end{vmatrix} + \begin{vmatrix} \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{23} & \varepsilon_{33} \end{vmatrix} \qquad (6)$$

$$J_2 = I_2 - \frac{1}{3} I_1^2 \qquad (7)$$

3. RESULTS AND DISCUSSION

The numerical experiments were designed to study the effects of the granular flow during the silo discharge. After concluding the simulations, it was determined that the nature of the granular flow was tubular as shown in Figure 3. Based on this, a cylindrical volume control was assigned in the region around the discharge hole to characterise the mechanical response of the assembly. Specifically, velocity profiles, stresses in the silo walls, average deformation and force chains among particles were evaluated.

The velocity profile identified after completing the simulation cases was identified as the funnel flow. This finding is associated with the friction coefficient difference between the granular matter (μ_{corn} = 0.51) and the walls of the silo (μ = 0.52). The response of the particle assembly near to the silo walls appear to achieve static equilibrium; shown in Figure 4. This occurs due to the difference between the friction coefficients of the silo walls and corn.

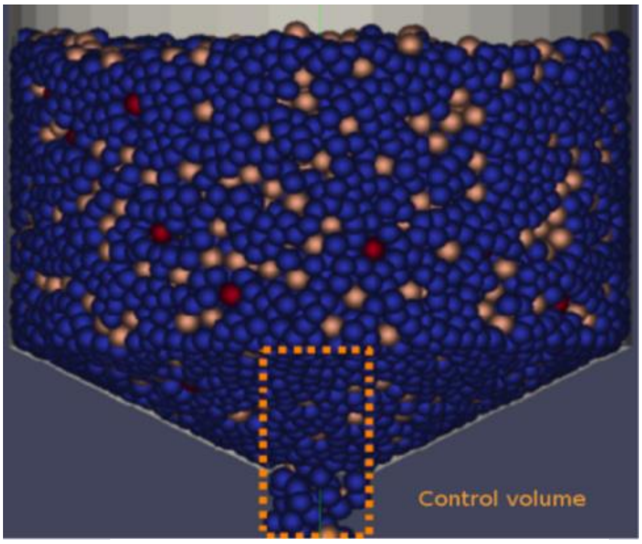


Figure 3. Funnel flow in the granular assemblies

3.1. Velocity profiles

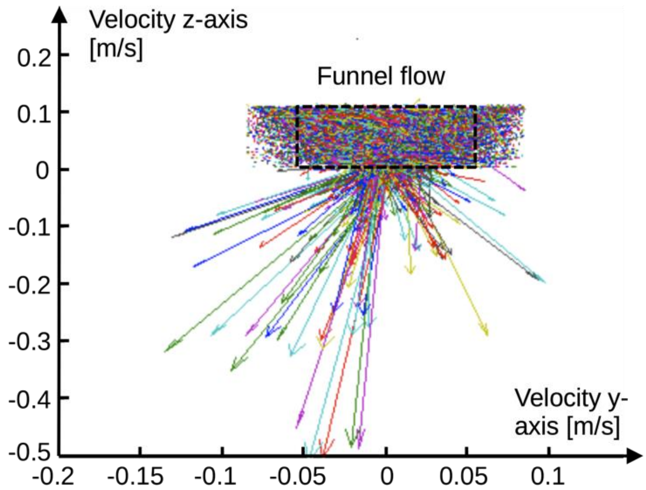


Figure 4. funnel flow during the discharge of the granular media stored in a silo with hopper

3.2. Shear stress and pressure in the walls

In Figure 5 is shown the comparison of the maximum shear stresses in the walls of the silo according to the hopper's angle. It is interesting to note that the repose angle (27°) appears to alter the trend in the shear stress magnitude. The maximum value occurred at this inclination (612 Pa, SD=140.1 Pa). For the cases where the chute's angle was over 27°, the shear stress decreased. This effect could be associated with the gravity and particle-wall friction in the hopper's boundaries. Apparently, the inclination of the discharging chute governs the magnitude of the shear stress in their walls.

The pressure in the silo's walls showed an increasing trend according to the inclination of the hopper's angle (Figure 6). The maximum pressure was achieved at a hopper's angle of 35° with an approximate pressure magnitude of 44.5 KPa, SD=8.9 KPa. The lowest pressure was identified in the silo with a hopper angle of 5° (P = 15.31 KPa, SD=3.8 KPa). This behaviour differs with the outcomes of the shear stress where the magnitude was modified after reaching a maximum value at the chute inclination corresponding to the granular material repose angle.

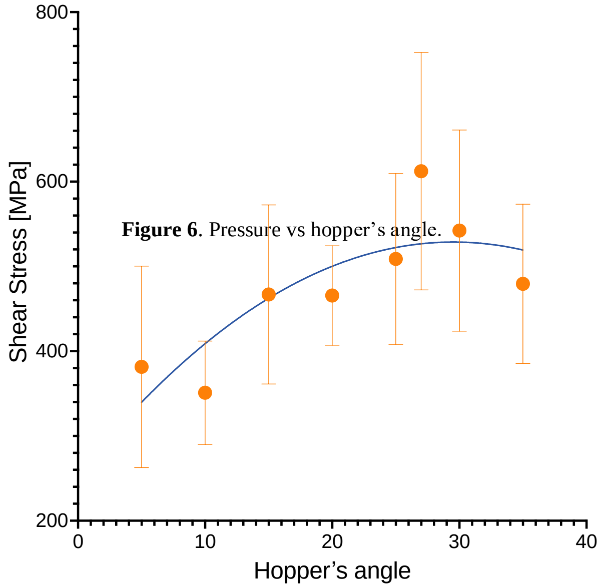


Figure 5. Shear stress vs hopper angle

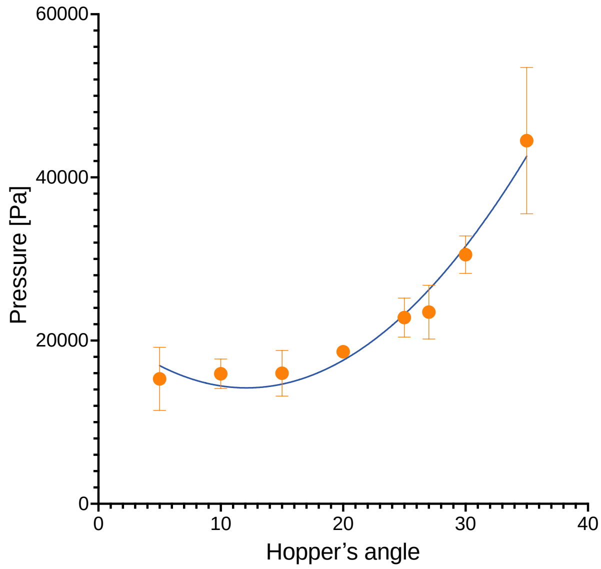


Figure 6. Pressure vs hopper angle

3.3. Average deformation in the stored granular media during its discharge.

As detailed in Section 2.6, the average deformation of the granular assemblies was quantified in terms of the principal invariant (I_1) and the second invariant of the strain deviator tensor (J_2). Both parameters were analysed in a control volume located in the zone of the discharging hole; shown in Figure 3. I_1 is useful to identify the volumetric changes (length alterations) of the granular set. On the other hand, the shape changes were measured by means of the second main invariant (J_2). Volumetric changes were the principal alterations registered during the discharge of the granular matter. The maximum I_1 value was identified at a hopper angle of 10° (4.88*10^{-5}, SD=8.65*10^{-5}) and at hopper's angle similar to the repose angle of corn (3.78*10^{-5}, SD=1.66*10^{-5}). The trend in the data displayed that the volumetric deformations increased until reaching the repose angle of corn (27°). After this, the magnitude of I_1 decreased for the other hopper's inclinations as shown in Figure 7. For the case of the shape changes, the maximum deformation occurred in the 5-degree chute inclination with approximately J_2 of -3*10^{-7}. Contrary to the hydrostatic strain (I_1), J_2 showed a decreasing tendency in the study cases, see Figure 8.

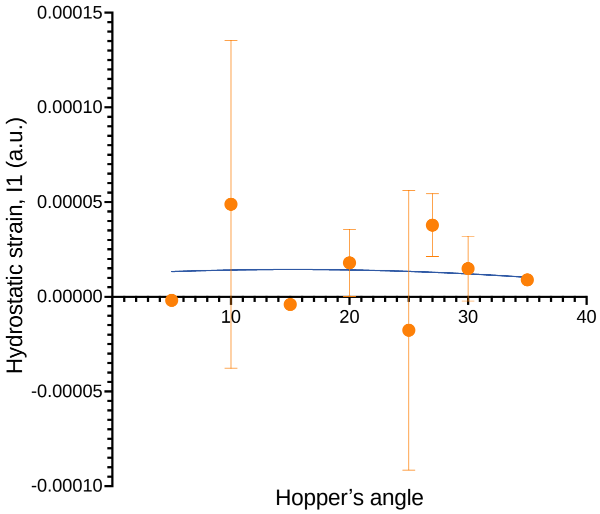


Figure 7. Hydrostatic strain, I1

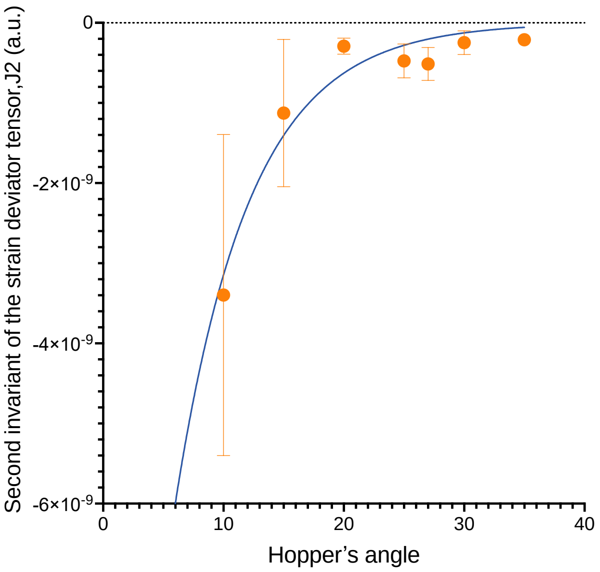


Figure 8. Second invariant of the strain deviator tensor J2

3.4. Force chains

The resulting force chains due to particles interaction entailed an increasing trend in their magnitudes until reaching the repose angle of corn (27°). The maximum reached value was 0.18 N. After this limit, the force chains reduce. This trend is similar to previous results such as shear stress and hydrostatic strain. The effects in the force chains are plot-ted in Figure 9. The force chains generated in the region surrounding the discharge hole are not strong enough to conform an arch and stop the corn flow.

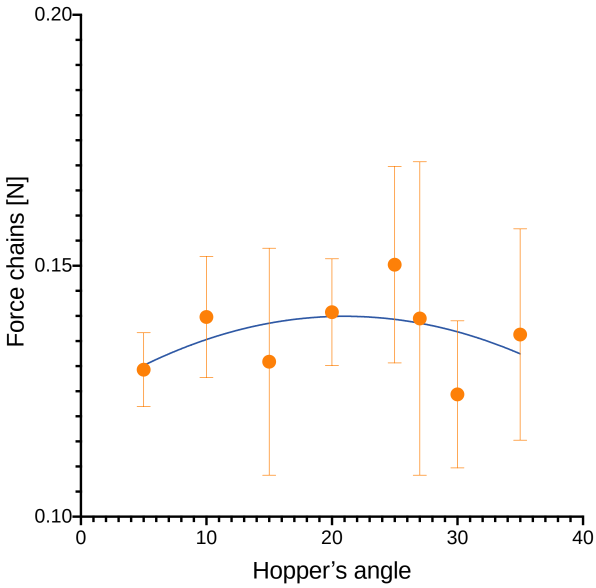


Figure 9. Force chains

3.5 Analysis of the results trend

An explanation to describe the results in the shear stress, deformation and force chains of the corn-granular assembly is shown in Figure 10. In this model, three grains of corn in contact were used to describe the influence of the hopper's angle in the granular assembly. After applying the sum of forces in each particle, Equations 8, 9 and 10 describe the contact forces between particles (F_n) in terms of the hopper angle (α), the friction coefficient (μ), the particle weight (W) and elasticity coefficient of the corn (k_n).

$$F_{AB}(\sin(30)+\mu\cos(30))+F_{BC}+\tan(\alpha)\,\mathrm{N} = \mathrm{W}\sin(\alpha) \quad (8)$$

$$(F_{AB}-F_{AC})(\mu\cos(30)+\sin(30)) = -\mathrm{W}\sin(\alpha) \quad (9)$$

$$\delta = \frac{F_n}{k_n} \quad (10)$$

After replacing the hopper angle (α) with values below (α<27°), same (α=27°) and above (α>27°), it was found that the α directly influences the micromechanical response of the corn-granular assembly. Shear stresses in the silo walls, force chains (F_n) and deformation (δ) (Equation 10) trends of the theoretical model agreed with the numerical results (Sections 3.3 and 3.4). The maximum magnitude of the force chains and

deformations were computed when α corresponds to the repose angle (α=27°).
Limitations with this theoretical model included the number of particles studied (only 3 particles), the diameter of the silo was not modified, and this could modify the magnitude of the contact forces among particles, and acceleration of particles was algo not included in the model.

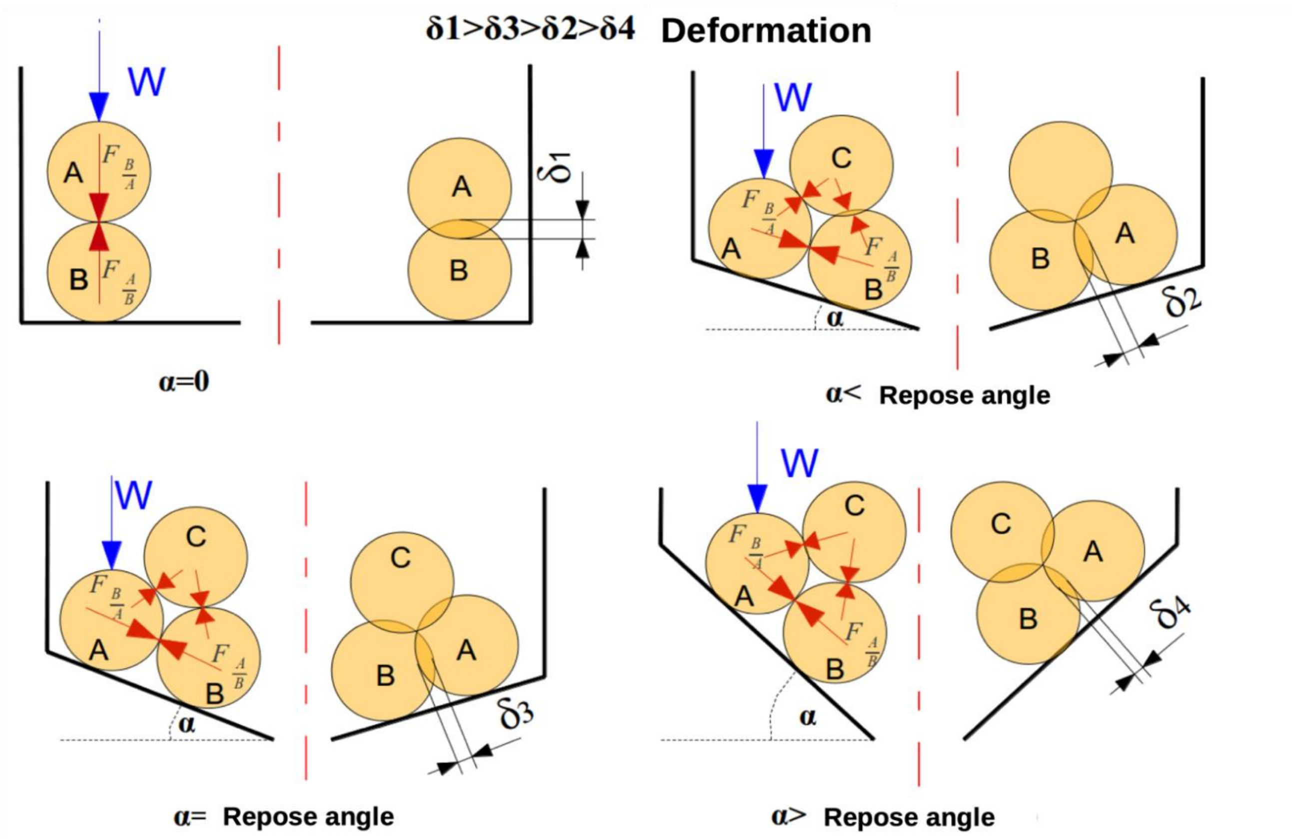


Figure 10. Description of the deformation of the corn-granular assembly based on different repose angles

4. CONCLUSIONS

The discrete element method (DEM) is a powerful modelling alternative to quantify the micro-mechanical effects in granular assemblies. This numerical technique is useful to identify interactions among particles which cannot be assessed by means of continuous formulations. The generated models in this research were useful to distinguish the effects of the silo geometry during its granular discharge.

In terms of the granular flow into the silo, it can be distinguished that the 'funnel flow' type appeared in all the studied cases. In order to identify this effect, a control volume in the discharging zone was used to differentiate how the particles are moving. These observations are consistent with the agreement that the coefficient of friction between particles is lower than the coefficient of friction between particles and walls.

It is worth mentioning that the hopper inclination, which design is based on the repose angle of the granular media, governs the micro-mechanical response of the granular flow. The baseline of comparison for each study case was the repose angle of corn (27°). A remarkable trend in the shear stress, hydrostatic strain and force chains was that after reaching a maximum value in a silo with a hopper angle equal to the repose angle, their magnitudes registered reduction. For the pressure and main invariant (J_2), there is an inverse relation with the chute inclination. In other words, whilst the hopper inclination increases, the pressure and J_2 tended to decrease.

REFERENCES

Andrade, J. E., Lim, K.-W., Avila, C. F., & Vlahinić, I. (2012). Granular element method for computational particle mechanics. *Computer Methods in Applied Mechanics and Engineering*, *241–244*, 262–274. https://doi.org/10.1016/j.cma.2012.06.012

Avila, C., & Andrade, J. (2012). Advances in multiscale modeling and characterization of granular matter. *Procedia IUTAM*, *3*, 157–171. https://doi.org/10.1016/j.piutam.2012.03.011

Baars, S. Van. (1995). *Discrete element modelling of granular materials* [Delft University of Technology]. https://repository.tudelft.nl/islandora/object/uuid%3A9ccd2776-6cd4-4536-b827-7feb49fda7bb

Babié, M. (1988). *Discrete Particle Numerical Simulation of Granular Material Behavior* [Clarkson University]. https://lin-web.clarkson.edu/~hhshen/teaching%20links/DEM%20document.pdf

Benyamine, M., Aussillous, P., & Dalloz-Dubrujeaud, B. (2017). Discharge flow of a granular media from a silo: effect of the packing fraction and of the hopper angle.

EPJ Web of Conferences, *140*(January), 03043. https://doi.org/10.1051/epjconf/201714003043

Boac, J. M. (2010). *Quality changes, dust generation, and commingling during grain elevator handling* [Kansas State University]. https://krex.k-state.edu/bitstream/handle/2097/2373/JosephineBoac2010.pdf?sequence=3&isAllowed=y

Brown, J. (2007). *Numerical analysis of silo discharge* [LUND University]. https://www.byggmek.lth.se/fileadmin/byggnadsmekanik/publications/tvsm5000/web5151.pdf

Cundall, P. A., & Strack, O. D. L. (1979). A discrete numerical model for granular assemblies. *Géotechnique*, *29*(1), 47–65. https://doi.org/10.1680/geot.1979.29.1.47

Elaskar, S., & Godoy, L. (2001). Simulación numérica del flujo de materiales granulares usando el concepto de estado crítico. *Revista Internacional de Métodos Numéricos Para Cálculo y Diseño En Ingeniería*, *17*(1), 19–36. https://upcommons.upc.edu/bitstream/handle/2099/3382/RR171B.pdf?sequence=1&isAllowed=y

Fernández, D. (2010). *Determinación de parámetros utilizados en las simulaciones d.e.m.* [Universidad Politécnica de Madrid]. https://oa.upm.es/10514/2/TESIS_MASTER_DANIEL_FERNANDEZ_LLANA.pdf

Goodman, M. A., & Cowin, S. C. (1972). A continuum theory for granular materials. *Archive for Rational Mechanics and Analysis*, *44*(4), 249–266. https://doi.org/10.1007/BF00284326

Gustafsson, G. (2008). *Simulation of iron ore pellets and powder flow using smoothed particle method* [Lulea University of Technology]. https://www.diva-portal.org/smash/get/diva2:990053/FULLTEXT01.pdf

Hill, P. (2012). PACT: A Course in Particle and Crystallization Technology. *2012 ASEE Annual Conference & Exposition Proceedings*, *June*, 25.1020.1-25.1020.8. https://doi.org/10.18260/1-2--21777

Ileleji, K. E., & Zhou, B. (2008). The angle of repose of bulk corn stover particles. *Powder Technology*, *187*(2), 110–118. https://doi.org/10.1016/j.powtec.2008.01.029

Ishkov, A. (2016). Energy-Efficient Devices for Transporting and Feeding Bulk Materials in the Construction Industry. *MATEC Web of Conferences*, *73*, 02019. https://doi.org/10.1051/matecconf/20167302019

Queteschiner, D., & Kloss, C. (2010). *Discrete Element Method*. http://calliope.dem.uniud.it/SEMINARS/ABSTRACT-SEMINARS/pres-queteschiner.pdf

Lira, C., & Pina, P. (2011). Granulometry on classified images of sand grains. *Journal of Coastal Research*, *1*(64), 1697–1701. https://www.jstor.org/stable/26482465

Mankoc, C., Janda, A., Arévalo, R., Pastor, J. M., Zuriguel, I., Garcimartín, A., & Maza, D. (2007). The flow rate of granular materials through an orifice. *Granular Matter*, *9*(6), 407–414. https://doi.org/10.1007/s10035-007-0062-2

Moya, M., Aguado, P. J., & Ayuga, F. (2013). Mechanical properties of some granular agricultural materials used in silo design. *International Agrophysics*, *27*(2), 181–193. https://doi.org/10.2478/v10247-012-0084-9

O'Sullivan, C. (2011). *Particulate Discrete Element Modelling* (1st ed.). CRC Press. https://doi.org/10.1201/9781482266498

Parafiniuk, P., Molenda, M., & Horabik, J. (2013). Discharge of rapeseeds from a model silo: Physical testing and discrete element method simulations. *Computers and Electronics in Agriculture*, *97*, 40–46. https://doi.org/10.1016/j.compag.2013.06.008

Rotter, J., Holst, J., Ooi, J., & Sanad, A. (1998). Silo pressure predictions using discrete-element and finite-element analyses. *The Royal Society*, *356*(1747), 2685–2712. https://doi.org/10.1098/rsta.1998.0293

Savage, S. B. (1979). Gravity flow of cohesionless granular materials in chutes and channels. *Journal of Fluid Mechanics*, *92*(1), 53–96. https://doi.org/10.1017/S0022112079000525

Tan, Y., Xiao, X., Zheng, J., Jiang, S., & Gao, W. (2016). *Effect of outlet diameter of cone-in-cone insert on silo flow pattern*. *32*, 82–87. https://doi.org/10.11975/j.issn.1002-6819.2016.19.011

Wang, Y., Lu, Y., & Ooi, J. Y. (2015). A numerical study of wall pressure and granular flow in a flat-bottomed silo. *Powder Technology*, *282*, 43–54. https://doi.org/10.1016/j.powtec.2015.01.078

Wieckowski, Z. (2003). Modelling of silo discharge and filling problems by the material point method. *Task Quarterly*, *4*(4), 22. https://journal.mostwiedzy.pl/TASKQuarterly/article/view/2182

Zhou, Y., Lagreé, P. Y., Popinet, S., Ruyer, P., & Aussillous, P. (2017). Experiments on, and discrete and continuum simulations of, the discharge of granular media from silos with a lateral orifice. *Journal of Fluid Mechanics*, *829*, 459–485. https://doi.org/10.1017/jfm.2017.543

Zuriguel, I., Garcimartín, A., Maza, D., Pugnaloni, L. A., & Pastor, J. M. (2005). Jamming during the discharge of granular matter from a silo. *Physical Review E - Statistical, Nonlinear, and Soft Matter Physics*, *71*(5). https://doi.org/10.1103/PhysRevE.71.051303

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