Two-Wheeled Inverted Pendulum Robot NXT Lego Mindstorms: Mathematical Modelling and Real Robot Comparisons

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Resumen: El objetivo de este artículo es desarrollar y comparar diferentes modelos de un robot tipo péndulo invertido de dos ruedas llamado NXT Lego Mindstorms. Dos modelos son desarrollados: un modelo completo no lineal y uno lineal obtenido mediante estimación por mínimos cuadrados. Las respuestas de ambos modelos son comparadas con la respuesta del robot real. El modelado de procesos es fundamental en la síntesis de controladores, ya que en su mayoría el diseño de controladores está basado en modelos matemáticos. Por lo tanto, en la medida que obtengamos mejores y más exactos modelos más fácilmente será derivar controladores que presenten un desempeño satisfactorio.

Palabras clave: modelado, validación, robot móvil, péndulo invertido, Lego Mindstorms, modelo no lineal.

Abstract: The aim of this paper is to develop and compare different models of a two-wheeled inverted pendulum robot Lego Mindstorms. Two models are developed, a complete nonlinear one, and a linear one based on least squares estimation. The responses of both models are compared with the response of the real robot. Processes modeling is fundamental in controllers' synthesis, since most of the controllers design are based on mathematical models. Therefore, if better and more accurate mathematical models are obtained more easily is the controller's synthesis for satisfactorily performance.

Keywords: modeling, validation, mobile robot, inverted pendulum, Lego Mindstorms, nonlinear model.

1. INTRODUCTION

The concept of balancing a robot is based on the inverted pendulum model idea. An inverted pendulum is an open loop unstable system with highly non-linear dynamics. The inverted pendulum problem is common in the field of control engineering. The uniqueness and wide application of technology derived from this unstable system has drawn interest of researchers and robotics enthusiasts around the world [1-3]. Therefore, it represents an ideal experiment for the design of classical and contemporary control techniques [3]. It has broad ranging applications from robotics to space rocket guidance systems [1].

Mobile robotics is a rapidly expanding field. In the near future, mobile robots could be used in a large number of applications, particularly in assisting and interacting with humans e.g. transporting materials around offices or hospitals, domestic activities, etc. The type of robot used in this paper is a mobile robot with a two wheeled inverted pendulum, known as the Two-Wheeled inverted Pendulum Robot NXT Lego Mindstorms. The robot has a body with two wheels for moving in a plane and a head similar to a human head. Two independent driving wheels are used for position control and for fast motion in a plane.

The educational platform Lego Mindstorms NXT 2.0 is a new generation in educational robotics that can assemble numerous mechanical configurations and also allows the possibility of programming in various languages (robot is programmed on RobotC.)

In Fig. 1 the Lego Mindstorms NXT platform is shown. It is configured as a two-wheeled inverted pendulum:

The main components are: 1) NXT brick which is the brain of Lego Mindstorms, 2) compass sensor which measures the angle of orientation (ϕ) and also estimates their speed ($\dot{\phi}$), 3) gyro sensor which measures the speed of inclination $\dot{\psi}$ and estimates their angle ψ and 4) electric actuators for both wheels, which contain angular position sensors (encoders) to measure the angular position θ of the wheels and estimate their speed $\dot{\theta}$. [4, 5].

Figure 1. The NXT Lego Mindstorms system.

There are some papers dedicated to mobile robots. Most of them describe control and educational tasks. For instances, Christian Sundin and FilipThorstensson [6]. This project consistsin designing and building a two wheeled upright robot. The robot was designed for use on display tables at exhibitions. It has visible components and features some functions, like the display and some sensors, whose task is to draw interest from the surroundings. It interacts in a small extent to the surroundings by using a distance sensor in combination with a temperature sensor which makes it possible to distinguish a living being from an object. The robot also has a bowl on top for carrying a load.

A mathematical model was made for simulations and to test and dimension the controllers for standing upright and for movement. Both a PID and a LQG-controller were implemented and tested on the robot as well as different filters [7]. In this paper is presented an approach to be used in teaching for undergraduate courses. Patete, Aguirre, and Sanchez [3] showed, the inverted pendulum modelled using the Lagrange method, and from it a reduced model is given for the no linear system model. They used a PID algorithm to control the Lego robot. Canal and Brunet [8], presented an educational framework based on the Lego Mindstorms NXT robotic platform used to outline both the theoretical and practical aspects of the Model Predictive Control theory.

The aim of this paper is the development and comparison of two different models of the robot described above. This process is crucial since most of the controller design is based on mathematical models. Thus, if better and more accurate mathematical models are obtained more easily is the controller's synthesis for satisfactorily performance. Two models are obtained, firstly a complete nonlinear model is developed, and secondly a linear model based on the least squares method is also presented.The performance of both mathematical models are compared with the real robot.

This work is divided as follows: an introduction is presented, second section shows the nonlinear model and linear model, in section 3 a comparison among models and real robot is presented, and finally, conclusions are written.

2. MODELLING OF TWO-WHEELED INVERTED PENDULUM ROBOT

In this section a nonlinear model and a linear model are detailed. It is considered for modelling purposes six state variables and two inputs.

2.1 Nonlinear Modelling of Two-Wheeled Inverted pendulum Robot

In order to obtain the nonlinear mathematical model of this system, the model as is given in [4] is going to be taken as a reference including the dynamic equations which describe the behaviour of the system.

From Fig. 2, the following angles are identified:

ψ: Body pitch angle.
θ: Angular position of the right and left wheel.
ϕ: Body yaw angle.

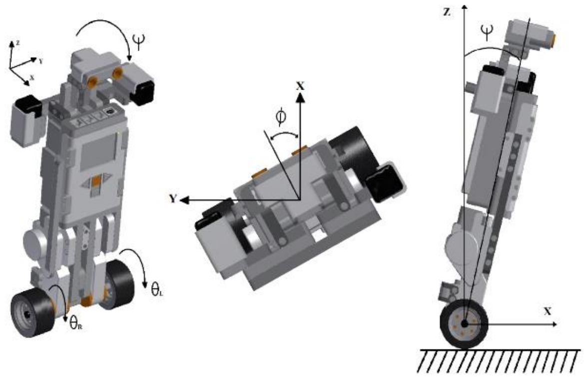

Figure 2. Lateral and top view of the system.

2.1.1 Dynamic equations of the system

By using the model as is shown in Fig.3, the Lagrange method, the kinetics energy translational and rotational, and the potential energy the forces as given in [4]:

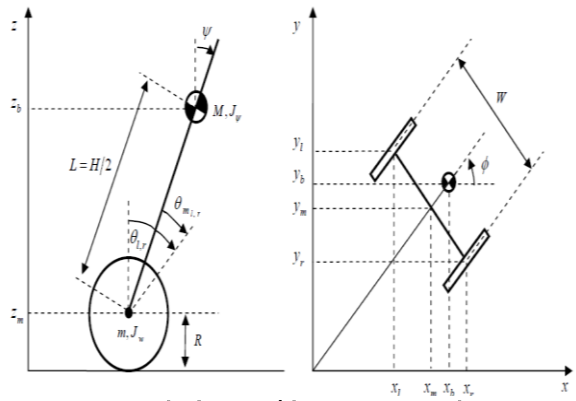

Figure 3. Front, top and side view of the NXT Lego Mindstorms inverted pendulum.

$$F_\theta = [(2m+M)R^2 + 2J_w + 2n^2 J_m]\ddot{\theta} + (MLR\cos\psi - 2n^2 J_m)\ddot{\theta} - MLR\dot{\psi}^2 \sin\psi \quad (1)$$

$$F_\psi = [MLRcos?\psi - 2n^2 J_m]\ddot{\theta} + (ML^2 + J_w + 2n^2 J_m)\ddot{\psi} - MgLsin\psi - ML^2\dot{\phi}^2 sin\psi\, cos\psi \quad (2)$$

$$F_\phi = [\frac{1}{2}mW^2 + J_\phi + \frac{W^2}{2R^2}(J_w + n^2 J_m) + ML^2 sin^2\psi]\ddot{\theta} + 2ML^2\dot{\psi}\dot{\phi}\, sin\psi cos\psi \quad (3)$$

Y Considering the torque of the DC motor, and the viscous friction, the forces in a general way could be expressed as follows:

$$(F_\theta, F_\psi, F_\phi) = (F_l + F_r,\ F_\psi,\ \frac{W}{2R}(F_r - F_l)) \quad (4)$$

$$F_l = nK_t i_l + f_m(\dot{\psi} - \dot{\theta}_l) - f_w\dot{\theta}_l \quad (5)$$

$$F_r = nK_t i_r + f_m(\psi - \dot{\theta}_r) - f_w\dot{\theta}_r \quad (6)$$

$$F_\psi = -nK_t i_l - nK_t i_r - f_m(\dot{\psi} - \dot{\theta}_l) - f_m(\dot{\psi} - \dot{\theta}_r) \quad (7)$$

The forces can be expressed by using the voltage of the DC motor as it is shown in (8), (9) and (10). [4]

$$F_\theta = \alpha(v_l + v_r) - 2(\beta + f_w)\dot{\theta} + 2\beta\dot{\psi} \quad (8)$$

$$F_\psi = -\alpha(v_l + v_r) + 2\beta\dot{\theta} - 2\beta\dot{\psi} \quad (9)$$

$$F_\phi = \frac{W}{2R}(v_l - v_r) - \frac{W^2}{2R^2}(\beta + f_w)\dot{\phi} \quad (10)$$

Where:

$$\alpha = \frac{nK_t}{R_m},\quad \beta = \frac{nK_t K_b}{R_m} + fm$$

2.1.2 Non-linear model of the system

Equating (1), (2), (3) and (8), (9), (10) the following expressions are obtained [9]:

$$[(2m+M)R^2 + 2J_w + 2n^2 J_m]\ddot{\theta} + (MLRcos\psi - 2n^2 J_m)\ddot{\psi} - MLR\dot{\psi}^2 sin\psi = \alpha(v_l + v_r) - 2(\beta + f_w)\dot{\theta} + 2\beta\dot{\psi} \quad (11)$$

$$[MLRcos\psi - 2n^2 J_m]\ddot{\theta} + (ML^2 + J_w + 2n^2 J_m)\ddot{\psi} - MgLsin\psi - ML^2\dot{\phi}^2\, sin\psi cos\psi = -\alpha(v_l + v_r) + 2\beta\dot{\theta} - 2\beta\dot{\psi} \quad (12)$$

$$[\frac{1}{2}mW^2 + J_\phi + \frac{W^2}{2R^2}(J_w + n^2 J_m) + ML^2\, sin^2\psi]\ddot{\phi} + 2ML^2\dot{\psi}\dot{\phi} sin\psi cos\psi = \frac{W}{2R}\alpha(v_r - v_l) - \frac{W^2}{2R}(\beta + f_w)\dot{\phi} \quad (13)$$

Finding $\ddot{\theta}$ in (12) and replacing in (11) the following is obtained:

$$\Delta = (MLR\, cos\psi - 2n^2 J_m)^2 - [(2m+M)R^2 + 2J_w + 2n^2 J_m](ML^2 + J_\psi + 2n^2 J_m)$$

$$\ddot{\psi} = \{[\alpha(v_l + v_r) - 2(\beta + f_w)\dot{\theta} + 2\beta\dot{\psi} + MLR\dot{\psi}^2 sin\psi](MLR\, cos\psi - 2n^2 J_m) - [(2m+M)^2 + 2J_w + 2n^2 J_m][MgLsin\psi + ML^2\dot{\phi}^2 sin\psi\, cos\psi - \alpha(v_l + v_r) + 2\beta\dot{\theta} - 2\beta\dot{\psi}]\}(\frac{1}{\Delta}) \quad (14)$$

Finding $\ddot{\psi}$ in (11) y replacing in (12) is obtained the following:

$$\ddot{\theta} = \{[MgLsin\psi + ML^2\dot{\phi}^2 sin\psi cos\psi - \alpha(v_l + v_r) + 2\beta\dot{\theta} - 2\beta\dot{\psi}](MLRcos\psi - 2n^2 J_m) - [ML^2 + J_\psi + 2n^2 J_m][MLR\dot{\psi}^2 sin\psi + \alpha(v_l + v_r) - 2(\beta + f_w)\dot{\theta} + 2\beta\dot{\psi}]\}(\frac{1}{\Delta}) \quad (15)$$

From (13), $\ddot{\phi}$ is obtained:

$$\ddot{\phi} = \frac{\frac{W}{2R}(v_r - v_l) - \frac{W^2}{2R^2}(\beta + f_w)\dot{\phi} + 2ML^2\dot{\psi}\dot{\phi}\, sin\psi\, cos\psi}{\frac{1}{2}mW^2 + J_\phi + \frac{W^2}{2R^2}(J_w + n^2 J_m) + ML^2 sin^2\psi} \quad (16)$$

2.1.3 Non-linear model of the system

Fig. 3 shows the front, top and side view of the inverted pendulum robot. The state vector is:

$$X = [\psi\ \dot{\psi}\ \theta\ \dot{\theta}\ \phi\ \dot{\phi}]^T = [x_1\ x_2\ x_3\ x_4\ x_5\ x_6]^T \quad (17)$$

The inputs vector is:

$$U = [v_l\quad v_r]^T = [u_1\quad u_2]^T \quad (18)$$

Where U represents the voltage control signals, u_2 for the right wheel and$u_1$for the left one. The output equation considers that all states are measurable. Then, the model in space state is given by [9]:

$$\Delta_e = (MLRcosx_1 - 2n^2 J_m)^2 - [(2m+M)R^2 + J_w + 2n^2 J_m](ML^2 + J_\psi + 2n^2 J_m)$$

$$\dot{x}_1 = x_2 \quad (19)$$

$$\dot{x}_2 = \{[\alpha(u_2 + u_1) - 2(\beta + f_w)x_4 + 2\beta x_2 + MLRx_2^2 sin\, x_1](MLRcosx_1 - 2n^2 J_m) - [(2m+M)R_2 + 2J_w + 2n^2 J_m][MgLsin\, x_1 + ML^2 x_6^2 sin\, x_1\, cos\, x_1 - \alpha(u_2 + u_1) + 2\beta x_4 - 2\beta x_2]\}(\frac{1}{\Delta_e}) \quad (20)$$

$$\dot{x}_3 = x_4 \quad (21)$$

$$\dot{x}_4 = \{[MgLsin\, x_1 + ML^2 x_6^2 sin\, x_1\, cos\, x_1 - \alpha(u_2 + u_1) + 2\beta x_4 - 2\beta x_2](MLRcosx_1 - 2n^2 J_m) - [ML^2 + J_\psi + 2n^2 J_m]$$

$$[MLRx_2^{\,2} sin\, x_1 + \alpha(u_2 + u_1) - 2(\beta + f_w)x_4 + 2\beta x_2]\}(\frac{1}{\Delta_e}) \tag{22}$$

$$\dot{x}_5 = x_6 \tag{23}$$

$$\dot{x}_6 = \frac{\frac{W}{2R}\alpha(u_2 - u_1) - \frac{W^2}{2R^2}(\beta + f_w)x_6 + 2ML^2 x_2 x_6 sin\, x_1 cos\, x_1}{\frac{1}{2}mW^2 + J_\phi + \frac{W^2}{2R^2}(J_m + n^2 J_m) + ML_2 sin^2 x_1} \tag{24}$$

From the previous equations we can notice that (θ y ϕ) do not affect to $\ddot{\psi}$ or $\ddot{\phi}$.

2.1.4Linear Modelling of Two-Wheeled Inverted pendulum Robot

The outputs have a linear behavior if they are seen from the state space. They have the following form:

$$y = f(x_1, x_2, \cdots, x_6, U_1, U_2) \tag{25}$$

Each output can be estimated by minimizing the square error in their parameters. The estimated outputs in each sample can be written as follows:

$$\begin{aligned}
\hat{y}1_{(1)} &= \hat{P}0_{10} + \hat{P}0_{11}x_1 + \cdots + \hat{P}0_{16}x_6 + \hat{P}0_{17}U_1 + \hat{P}0_{18}U_2 \\
\hat{y}2_{(1)} &= \hat{P}0_{20} + \hat{P}0_{21}x_1 + \cdots + \hat{P}0_{26}x_6 + \hat{P}0_{27}U_1 + \hat{P}0_{28}U_2 \\
&\vdots \\
\hat{y}n_{(1)} &= \hat{P}0_{n0} + \hat{P}0_{n1}x_1 + \cdots + \hat{P}0_{n6}x_6 + \hat{P}0_{n7}U_1 + \hat{P}0_{n8}U_2 \\
&\vdots \\
\hat{y}n_{(k)} &= \hat{P}0_{k} + \hat{P}0_{k1}x_1 + \cdots + \hat{P}0_{k6}x_6 + \hat{P}0_{k7}U_1 + \hat{P}0_{k8}U_2
\end{aligned} \tag{26}$$

According to [5] in order to obtain the reduced order estimation $\hat{P}0_0$ will be 1, and the number of samples [k] has to be bigger than the number of outputs in order to obtain an acceptable estimation, so the estimated outputs can be rewritten as:

$$\hat{y}(k) = X_g * \hat{P}0 \tag{27}$$

The error between the real outputs and the estimated ones in each sample is:

$$e(k) = y(k) - \hat{y}(k) \tag{28}$$

Replacing the estimated output expression in the equation (28):

$$e(k) = y(k) - X_g * \hat{P}0 \tag{29}$$

In order to minimize the square error of the parameters $\hat{P}0$ a quadratic performance index needs to be minimized as well. The index is [3]:

$$J = \|e(k)\|^2 \tag{30}$$

This is equal to:

$$J = \frac{1}{2} * e^T * e \tag{31}$$

The minimum square error is obtained with the partial derivative of J as follows:

$$\frac{\delta J}{\delta \hat{P}0} = \frac{\delta((y(k) - X_g * \hat{P}0)^T * (y(k) - X_g * \hat{P}0))}{2\delta \hat{P}0} \tag{32}$$

Considering these properties:

$$\frac{d(Ax)}{dx} = A^T; \frac{dx^T(Ax)}{dx} = Ax + A^T x; \frac{dx^T A}{dx}$$

Where A is a matrix, and the estimated parameters will be:

$$\hat{P}0 = (X_g^{\,T} * X_g)^{-1} * X_g^{\,T} * y(k) \tag{33}$$

Therefore, the least square method provides a linear model of the inverted pendulum, linearized around an equilibrium point. X_g must be a full rank matrix. Moreover, inside $\hat{P}0^T$ the matrices A and B, of the linear model, are contained; its first column is an independent term that tends to zero.

With 2000 samples and a sampling time of 10 [ms] the model for the NXT Lego Mindstorms Two- Wheeled Inverted Pendulum linearized by the least square method is:

$$A = \begin{bmatrix} 1.0037 & 0.0092 & 0 & 0.0008 & 0 & 0 \\ 0.6347 & 0.9004 & 0 & 0.1034 & -0.0001 & 0.002 \\ -0.01 & 0.0071 & 1 & 0.0029 & 0 & 0 \\ -1.09 & 0.8795 & 0 & 0.1103 & 0 & -0.0002 \\ 0.0002 & 0 & 0 & 0 & 1 & 0.0031 \\ 0.0104 & -0.0009 & -0.0003 & 0.0009 & -0.0007 & 0.0493 \end{bmatrix} \tag{34}$$

$$B = \begin{bmatrix} -0.0008 & -0.0008 \\ -0.1005 & -0.1005 \\ 0.0069 & 0.0069 \\ 0.8647 & 0.8647 \\ 0.0019 & -0.0019 \\ 0.2637 & -0.2637 \end{bmatrix} \tag{35}$$

3. MODELS AND ROBOTCOMPARISONS

For comparison purposes the response of the real system is comparedagainst the nonlinear model and the linear one. A linear quadratic controller is implemented which is detailed below.

3.1 Implementation of the discrete linear quadratic regulator (LQR)

To validate the obtained models an optimal feedback control LQR is performed. The linear system is given by:

$$x(k+1) = Ax(k) + Bu(k) \qquad (36)$$

Which must be transformed from a state $x(0)=x_0$ to another state $x(N)=0$, by minimizing a performance index:

$$J = \sum_{k=0}^{N} [x^t(k)Qx(k) + u^t(k)Ru(k)] \qquad (37)$$

To implement the optimal controller LQR from the linearized system and minimizing the performance index as is in Eq. (25), the objective is to find an optimal relationship between the system variables and the control action in this way, the matrices Q and R will be:

$$Q_x = \begin{bmatrix} 800 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 80 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} 10000 & 0 \\ 0 & 10000 \end{bmatrix} \qquad (38)$$

Where the optimal gain vectors for feedback obtained are:

$$Ku_1 = [-85.2,-11.5,-1.2,-1.3,-1.7,-0.005]$$
$$Ku_2 = [-85.18,-11.5,-1.2,-1.3,1.7,0.005]$$

3.2 Tests and results

Inverted pendulum models, both linear and nonlinear, cannot be validated directly since it is an open loop unstable system. It is not possible to analyze the process response using a step change due to the robot cannot stay balanced unless a controller be used. In order to test both mathematical models against the real robot a LQR controller has been implemented in the Lego Mindstorms NXT inverted pendulum.

To test the mathematical models, an external disturbance is considered, a constant force to keep a deviation of 0.1 rad. in (ψ) angle. Next figures show the real response (robot) and the mathematical models responses for ψ,θ,ϕ, and also for speeds $\dot{\psi},\dot{\theta},\dot{\phi}$.

To analyze the models, some disturbances are applied. They are detailed below:

An external force is exerted on the front of the robot; in Fig. 4 the sequence of the dynamic behavior of the robot is shown.

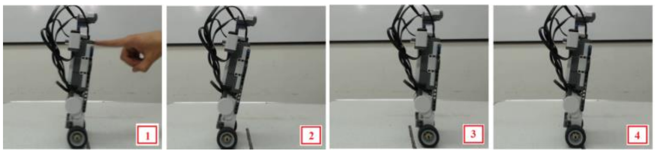

Figure 4. Dynamic behavior of the robot with an external front force.

Fig. 5 shows all the responses for de ψ angle. In spite that the dynamical shapes are similar, the settling time are not equal. The linear model response is faster than the others two.

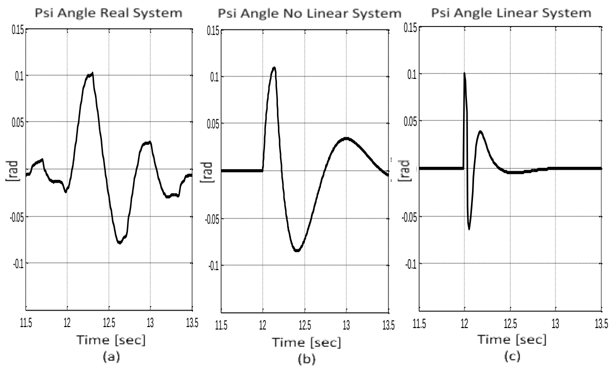

Figure 5. ψ angle. Robots and models responses

In Fig. 6 are depicted the speed of the ψ angle for robot and mathematical models.

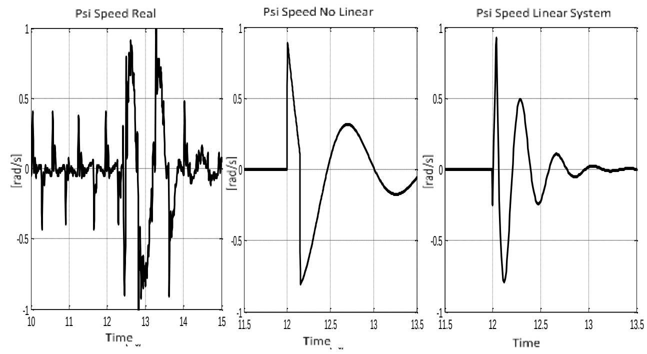

Figure 6. $\dot{\psi}$ for Robot and models

When an upset occurs, it affects the angle θ that represents the angular displacement of the tires in an instant of time. The angular displacement of the wheels can be seen in Fig. 7.

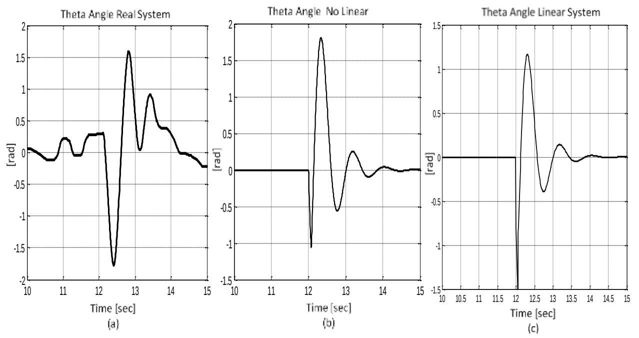

Figure 7. θ for robot and mathematical models.

The speed of the tires can be seen in Fig.8.

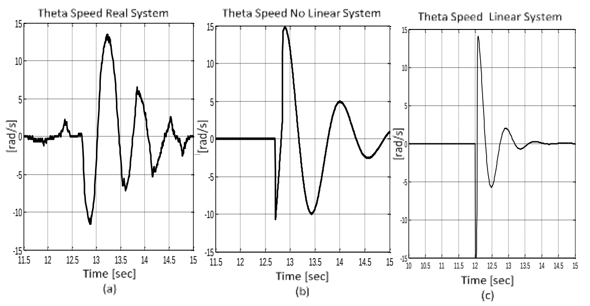

Figure 8. $\dot{\theta}$ for robot and models

The disturbance does not affect the orientation of the system; it is reflected in the angle ϕ. The ϕ angle differences for real system and mathematical models are negligible as is shown in Fig. 9.

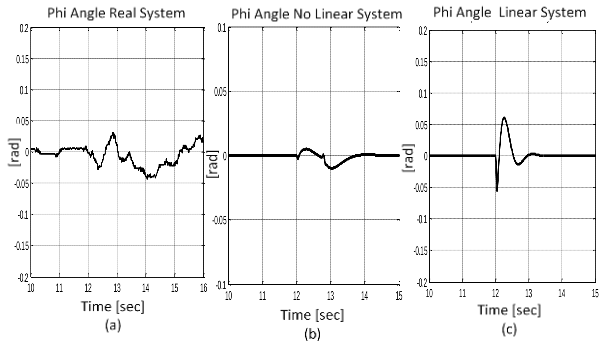

Figure 9. ϕ Comparison

Fig. 10 shows that changes in speed for ϕ angle is also minimal.

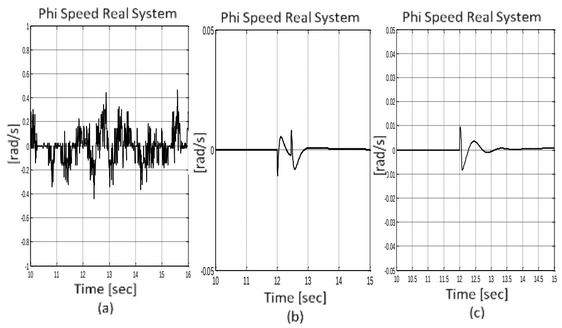

Figure 10. $\dot{\phi}$ Comparison

An external force is exerted on the back of the robot .The disturbance is considered, a constant force to keep a deviation of -0.12 rad. in (ψ) angle; in Fig.11 the sequence of the dynamic behavior of the robot is shown.

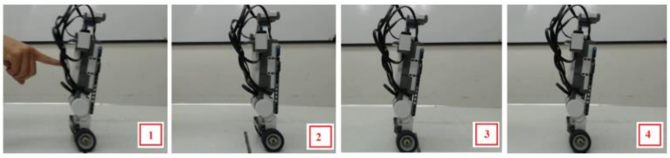

Figure 11. Dynamic behavior of the robot with an external back force.

The inclination angle (ψ) under the presence of the external force applied on the back of the robot, behaves as it is presented in Fig. 12 the plot (a) represents the real system, (b) the nonlinear model, and (c) the linear model.

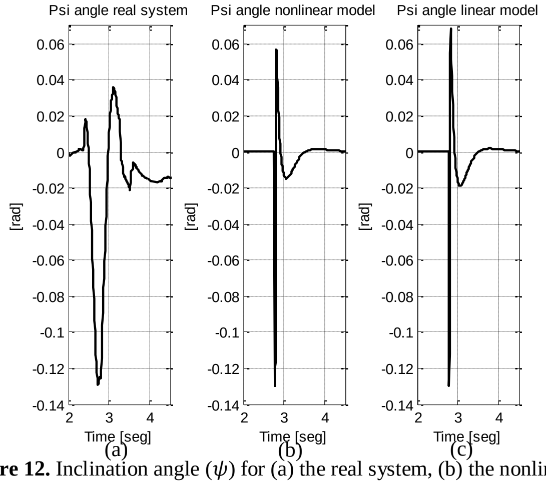

Figure 12. Inclination angle (ψ) for (a) the real system, (b) the nonlinear model and (c) the linear model under an external back force.

Fig. 13 shows the angular position of the wheels (θ) when the external back force is applied. (a) The real system, (b) the nonlinear model, and (c) the linear model.

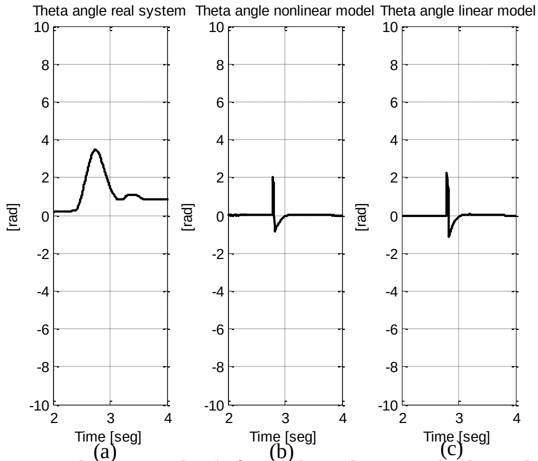

Figure 13. Inclination angle (θ) for (a) the real system, (b) the nonlinear model and (c) the linear model under an external back force.

3. CONCLUSIONS

This paper has shown the development of two models for the NXT Lego Mindstorms. A nonlinear one which takes into account the motor and robot dynamics, and a linear one based on the least square method.

The responses for both linear and non linearmodels, cannot be compared directly against the robot response, in order to do that a LQR should be used to keep balanced the robot, since it is an open loop unstable system,

The results indicate that the ψ and θ responses for the real system are similar in shape than the presented by the models, but some model parameters adjustments should be done in order to get closer settling times for the models with respect to the real robot.

The ϕ angle differences for real system and mathematical models are negligible.

Both models can be used for controller’s synthesis, but if the linear one is used, it is recommendable to propose a robust controller scheme.

NXT Lego Mindstorms is a good tool for teaching and learning activities in control area.

ACKNOWLEDGMENT

Oscar Camacho thanks to PROMETEO Project of SENESCYT, Republic of Ecuador, for its support for the realization of this work.

REFERENCES

[1] A. N. K. Nasir, M. A. Ahmad, R. M. T. Raja Ismail, *"The Control of a Highly Nonlinear Two-wheels Balancing Robot: A Comparative Assessment between LQR and PID-PID Control Schemes"*, World Academy of Science, Engineering and Technology, Vol.: 4 2010.pp 176-181.

[2] J. Aracil, F. Gordillo, *"El Péndulo invertido, un desafío para el control no lineal"*.RIAI, Vol. 2. 2005, pp. 8-19.

[3] A.Patete, I. Aguirre, H. Sánchez, *"Control de un péndulo invertido basado en un modelo reducido"*, Revista INGENIERÍA UC, vol. 18, núm. 1, enero-abril, 2011, pp. 12-22.

[4] Y. Yamamoto. *"Nottaway-GS Model-Based Design - Control of self-balancing two-wheeled robot built with LEGO Mindstorms NXT"* - http://www.mathworks.com/matlabcentral/fileexchange/loadFile.do?objectId=13399&objectType=file. 2008.

[5] A. Jiménez, B. Mohammed Al-Hadithi, ,L. Alonso, *"Linear Quadratic Regulator Based Takagi-Sugeno Model for Multivariable Nonlinear processes"*,17th International Conference on System Theory, Control and Computing (IICSSTCC), Joint Conference SINTES 17, SACCS 13, SIMSIS 17, At Sinai, Romania. 2013.

[6] C.Sundin, F. Thorstenson. *"Autonomous balancing robot: Design and construction of a balancing robot"*. Master of Science Thesis. Chalmers University Of Technology. Goteborg, Sweden, 2012.

[7] A. Cruz-Martin, J.A. Fernandez-Madrigal, C. Galindo, J.Gonzalez-Jimenez, C. Stockmans-Daou, J.L.Blanco. *"A Lego MidstornsNtx Approach for Teaching at Data Acquisition. Control Systems Engineering And Real-Time Systems Undergraduate Courses"*. Computers and Education. 2012.

[8] M.Canale, S.CasaleBrunet. *"A Lego Mindstorms NXT experiment for Model Predictive Control education"* 2013 European Control Conference (ECC) July 17-19, 2013, Zürich, Switzerland.

[9] M. Herrera.*"Modelado Discreto y Control Óptimo de Sistemas No Lineales Multivariables y su Aplicación a un Péndulo Invertido utilizando Lego Mindstorms"*,Master of ScienceThesis, Universidad Politécnica de Madrid, España 2014