

Methodologies and Programming Development of Applications on Modeling, Identification and Simulation

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Resumen: *La modelación, simulación e identificación de prototipos permiten realizar predicciones acertadas del comportamiento de los sistemas, además de hallar las expresiones matemáticas tales como funciones de transferencia, ecuaciones diferenciales y variables de estado necesarias para la implementación de controladores, sin necesidad de la implementación de los prototipos reales, lo que permite simular en un entorno similar las perturbaciones o simplemente observar su respuesta en condiciones normales. Este trabajo presenta la modelación de varios sistemas, utilizando diversos métodos sean éstos directos, de redes o variacional, además del desarrollo de metodologías enfocadas al desarrollo de casos de estudio, a través de los cuales se implementa cinemática inversa, transformaciones homogéneas, métodos de Newton-Euler, Euler-Lagrange y otros que facilitan la simulación de sistemas a través de Matlab.*

Palabras clave: *Robótica, Modelación, Simulación, Identificación, Procesos.*

Abstract: *Modeling, simulation and identification of prototypes allow people to make right predictions about the behavior of systems also finding mathematical expressions such as transfer functions, differential equations and state variables needed to control implementation without being necessary to implement real prototypes which allow to simulate in a similar environment as the one with perturbation or simply observe the dynamical response in normal conditions. This paper is focused on modeling various systems, using diverse modeling methods as direct, net method or variational, also the development of methodologies dedicated on study cases developing through which is implemented inverted kinematics, homogenous transformations, Newton-Euler method, Euler-Lagrange method and some other methods that makes it simple to run system simulations on Matlab.*

Keywords: *Modeling, Identification, Simulation, Robotics, Process.*

1. INTRODUCTION

Modeling, simulation and identification of prototypes are a fundamental part of controllers design, obtained by mathematical modeling expressed as state variables, transfer functions and differential equations. From control system focusing, the goal of modeling is to find a system's analytic model which allows designing the right controller to close the system's loop to implement a correct dynamic system's operation, saving resources and let system quality get better. This paper introduces different methods of modeling that makes it easier to obtain its state variables, transfer function and differential equations. These methods [1] use interconnection restrictions to flux and effort, theory of electrical nets by applying node matrix equations and interconnection restrictions based on Lagrangian calculation.

Simulation is useful due to mathematical problem solutions in systems which analytic solution is unknown or in mathematical processes where the solution is complex when trying to calculate the analytic solution without any computational device. The simulation allows finding solutions to problems where experimentation is expensive and allows the investigation of possible control strategies, while providing a more realistic replica of a system.

Presenting a modeling and identification software allows obtaining the system answers according to the input specifications. Identification is acquired from experimental data by applying several iterations applied to the approximate equation of the line with nearby points and neglecting the experimental points coming out of the expected range of measurements.

2. DEVELOPMENT OF METHODOLOGIES FOR MODELING AND IDENTIFICATION

A model is a simplified representation of a system which answers to questions about the specified system are obtained without using experimentation. For identification, reaction curve is used for obtaining steady-state value of the system, this value is obtained by performing a measurement of the process and make a curve with the input and output values over time.

2.1 Direct Method

The procedure of modeling by using the direct method lets select the variables involved in a system, whether it is physical, electrical, mechanical, fluid, etc. The direct method allows the visualization of the direction of the forces applying physics by Newton's laws (Figure 1).

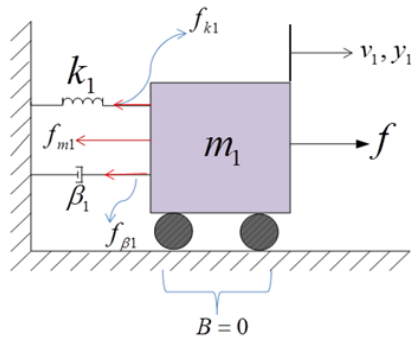


Figure 1: Mechanic System.

The system of Figure 1 is used for analysis by the direct method, for which it has several steps to obtain its differential equations and state variables.

- (1) Write a free body diagram, considering forces that act over it.

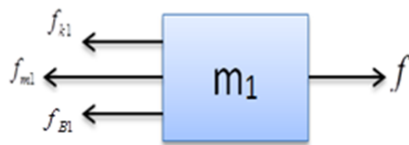


Figure 2: Free body diagram.

- (2) Applying Newton's second law, which intervenes the sum of the forces acting on the body, in the system acts four forces and are represented according to the direction of the mass 1.

f_{k1} = force produced by the spring.
 f_{B1} = force produced by the buffer.
 f_{m1} = reaction effort produced by the action of force.
 f = force produced by action.

- (3) Write the sum of forces accordingly to the body free diagram.

$$\sum f = 0$$

$$f_{m1} + f_{B1} + f_{k1} = f$$

$$\begin{cases} f_{m1} = m_1 a_1 = m_1 \dot{v}_1 = m_1 \ddot{y}_1 \\ f_{B1} = B_1 v_1 = B_1 \dot{y}_1 \\ f_{k1} = k_1 y_1 \end{cases}$$

$$m_1 \ddot{y}_1 + B_1 \dot{y}_1 + k_1 y_1 = f \quad (1)$$

- (4) Write the state variables according to the input and output applied to the system, in this case the system input is the action force f and output variables are the displacement and speed.

$$\begin{bmatrix} y_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad \begin{matrix} u = f \\ y = y_1 \end{matrix}$$

- (5) To describe each of the variables to obtain the matrices corresponding to the state variables.

v_1 = mass speed 1 and equivalent to the first derivative of displacement and equal to

$$\frac{dy_1}{dt}$$

$y_1 = x_1 =$ movement performed by the mass 1

$v_1 = x_2 = \dot{x}_1 =$ mass 1 speed

- (6) Replace the state variables in equation 1 for the state matrix.

$$m_1 \ddot{y}_1 + B_1 \dot{y}_1 + k_1 y_1 = f$$

$$m_1 \dot{x}_1 + B_1 x_2 + k_1 x_1 = u$$

$$\dot{x}_2 = -\frac{k_1}{m_1} x_1 - \frac{B_1}{m_1} x_2 + \frac{1}{m_1} u$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k_1}{m_1} & -\frac{B_1}{m_1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m_1} \end{bmatrix} u$$

$$y = (1 \ 0) \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad D = 0 \quad (2)$$

Equation 2 represents the matrix of variable states resulting A, B, C, D.

$$A = \begin{bmatrix} 0 & 1 \\ -\frac{k_1}{m_1} & -\frac{B_1}{m_1} \end{bmatrix} = \text{State Matrix.}$$

$$B = \begin{bmatrix} 0 \\ \frac{1}{m_1} \end{bmatrix} = \text{Input Matrix.}$$

$$C = (1 \ 0) = \text{Output matrix.}$$

$$D = 0 = \text{Zero matrix or direct transition matrix.}$$

2.2 Nets method





The method of generalized nets or nets is formed by the interconnection of elements and widespread sources that represent different kinds of physical elements and variables

involved in it allows obtaining quantities that describe the dynamic behavior of the same system. An electrical equivalent from a mechanical system benchmarks required for processing and proper connection. It is necessary to identify whether the variables are transvariables or pervariables.

- (1) Transvariables: those variables that require two points to be measured and are obtained through resistors, inductors or capacitors.
- (2) Pervariables: those variables that are propagated by the elements and for which measurement is required only one point.

Table 1 is analyzed considering the mechanical, fluid and electrical equivalent that facilitates the resolution of problems that arise around the modeling of systems.

Table 1: Mechanical and electrical analogies.

Mechanical Element		Electronic Element		Representation
Symbol	Amount	Symbol	Amount	
f	Force	f	Current Source	
M	Mass	C	Capacitor	
B	Shock Absorber	R	Resistance	
K	Spring	L	Inductor	

The mechanical system shown in Figure 2 is transformed to an electrical system through respectively equivalents according to Table 1. Observing the elements of the system and performing a comparison table to facilitate the transformation of electrical into mechanical elements, in this case the system comprises a damper and a spring supported mass through these elements to a reference system which the wall and the floor. which is subject of the aforementioned elements.

The characteristics of the processed electrical mechanical elements are deduced from this comparison:

$$e = \text{effort} = v = \text{velocity}$$

$$f = \text{force} = \text{flux} = i$$

$$\text{Input} = f = u$$

$$\text{Output} = y = \text{position}$$

$$v_i = L \frac{di}{dt}$$

$$\int v_i dt = \int L \frac{di}{dt} dt$$

$$e_a = \int v_i dt = x = Li = Lf$$

$$f = \frac{1}{L} x = kx$$

$$L = \frac{1}{k}$$

Force produced by the buffer $f_B = B\dot{v}$ goes like $1/B$

$$\frac{v}{f} = \frac{1}{B} = \frac{\text{effort}}{\text{flux}} \therefore \text{It is equivalent to } \frac{V}{I} = R$$

$$R = \frac{v}{f} = \frac{1}{B}$$

Construction begins with the equivalent circuit for the main mass of the system as it is transformed into a grounded capacitor mentioned, we consider each point mass or speed for the circuit node, in this case comes only a node that connects the elements in the system. It is recommended that in order of appearance in the circuit elements with reference to the capacitor referred to ground, the coil must be placed to the left of it and the resistor on the right side of the capacitor.

The external force applied to the system acts as a current source placed in the same direction the force is applied, in this case from the bottom up, allowing equivalent construct the network diagram.

Set the direction of the currents in accordance with basic knowledge of electric circuits, retained the name placed on the mechanical system to not confuse the nomenclature used.

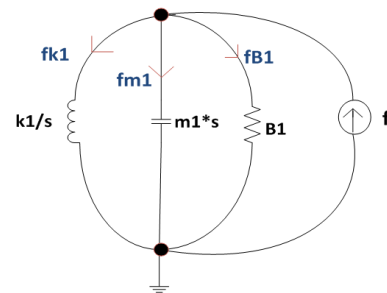


Figure 3: Net diagramat "s" domain.

Figure 3 represents the equivalent system with an inertial reference network and appropriately placed sense currents to observe the effect thereof and building or the respective equations for each system.

Applying law meshes and nodes required to obtain the equations.

$$f_{k1} + f_{m1} + f_{B1} = f \tag{3}$$

Write the state variables according to the input and output applied to the system, in this case the system input is the action force and the output variables are the displacement and the speed.

$$\begin{bmatrix} y_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \begin{bmatrix} u \\ y \end{bmatrix} = \begin{bmatrix} f \\ y_1 \end{bmatrix} \tag{4}$$

It is necessary to describe each of the variables to obtain the matrices corresponding to the state variables.

$v_1 =$ mass 1 speed and equivalent to the first derivative of displacement and equal to $\frac{dy_1}{dt}$
 $y_1 = x_1 =$ displacement made by mass 1
 $v_1 = x_2 = \dot{x}_1 =$ Mass 1 speed

Replace the state variables in equation 3 for the state matrix.

$$\begin{aligned}
 m_1 \ddot{y}_1 + B_1 \dot{y}_1 + k_1 y_1 &= f \\
 m_1 \dot{x}_2 + B_1 x_2 + k_1 x_1 &= u \\
 \dot{x}_2 &= -\frac{k_1}{m_1} x_1 - \frac{B_1}{m_1} x_2 + \frac{1}{m_1} u \\
 \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ -\frac{k_1}{m_1} & -\frac{B_1}{m_1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m_1} \end{bmatrix} u \\
 y &= (1 \ 0) \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad D = 0
 \end{aligned} \tag{5}$$

The equation 4 represents state variable matrix where we have A, B, C and D.

$$\begin{aligned}
 A &= \begin{bmatrix} 0 & 1 \\ -\frac{k_1}{m_1} & -\frac{B_1}{m_1} \end{bmatrix} = \text{State matrix} \\
 B &= \begin{bmatrix} 0 \\ \frac{1}{m_1} \end{bmatrix} = \text{Input matrix} \\
 C &= (1 \ 0) = \text{Output matrix} \\
 D &= 0 = \text{Direct transition matrix or zero matrix}
 \end{aligned}$$

Obtain the transfer function of the system which applies node matrix equations.

$$[Y] * [V_n] = [I] \tag{6}$$

Equation 6 represents the admittance in the system by the product of node voltage equals to the source which supplies power to the system.

$$\begin{aligned}
 [Y] * [V_n] &= [I] \\
 \left[\frac{k_1}{s} + m_1 s + B_1 \right] V_1 &= I = f \\
 \frac{V_1(s)}{F(s)} &= \frac{OUTPUT}{INPUT} = \frac{1}{\left[\frac{k_1}{s} + m_1 s + B_1 \right]} \\
 \frac{V_1(s)}{F(s)} &= \frac{s}{m_1 s^2 + B_1 s + k_1} = G_1(s)
 \end{aligned} \tag{7}$$

Correctly applying the equivalent net for mechanical systems and other fluids readily to write differential equations, state variables and transfer function posed systems without considering free body diagrams, physical laws and direction of forces applied.

2.3 Variational method

The mathematical models of physical systems (mechanical, electrical, etc.) can be derived from energy considerations without applying Newton's laws or Kirchhoff's laws. The mechanical equations of motion feature need to be derived, by using for the potential and kinetic energy there of.

In deriving the equations of motion for a complicated mechanical system, it should be done using two different methods (one based on Newton's second law and the other by considering energy acting on the system). The Lagrange method is a successful appeal to derive equations for this system class.

To derive the Lagrange equations of motion is necessary to define the generalized coordinates and the Lagrangian, to establish the principle of Hamilton [1]. Simplified model of a manipulator without considering the moment of inertia of the masses of each end of the robotic arm is considered. It disregards the moments of inertia for an idealized model of the system and easily to deduce the mathematical model.

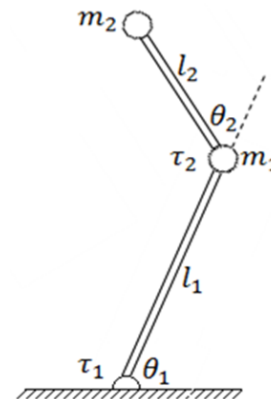


Figure 4: Simplified manipulator.

To apply the variational method is necessary to consider the initial conditions:

- Rotation of the mass schedule.
 - Set point mass 2, the floor is subject to where the first link, the reference is moved to the second link.
- (1) Performing speed diagram to determine the net rate at which the mass 2 moves, it is necessary to move in the same diagram references to relate vectors and complementary and supplementary angles.

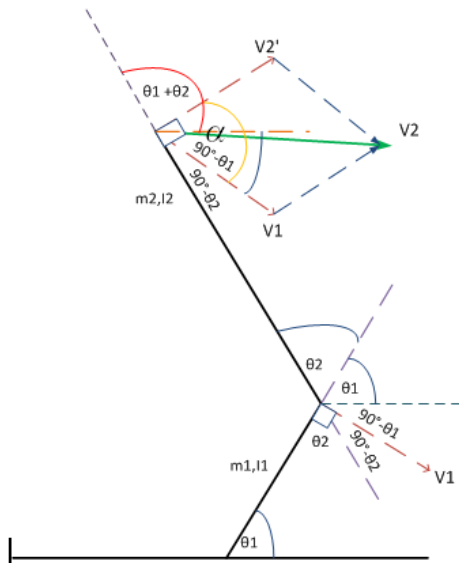


Figure 5: Relative speed diagram.

- (2) Performing the sum of the angles corresponding to the mass such that two have a supplementary angle, the sum is equals to 180° , and thus obtain the equivalent angle α .

$$90 + \alpha + 90 - \theta_2 = 180$$

$$\alpha = \theta_2$$

- (3) Applying cosine law to determine v'_2 once found the equivalent in terms of known angles relative speed of the second link is a function of the angle 1 and speed $2\theta_2$:

$$v'^2_2 = v^2_1 + v^2_2 - 2v_1v_2 \cos(180 - \theta_2)$$

$$v'^2_2 = v^2_1 + v^2_2 + 2v_1v_2 \cos(\theta_2) \quad (8)$$

- (4) Selecting generalized coordinates, these coordinates are independent and are necessary to describe the motion of the system, the number of variables depends on the degrees of freedom possessed by the system. In this case, are two coordinates: θ_1, θ_2 .
- (5) Selecting variational coordinates, these are the variables that place restrictions and permissible variations must be allowed throughout the operation of the: θ_1, θ_2 .
- (6) Calculate the Lagrangian, co-energy and co-contained, it is necessary to obtain the kinetic and potential energy of the system in a single expression that allows to apply the Euler-Lagrange equation based on terms of work and energy, all shares not generate a job are not considered for the system in this method.

$$L = U^* - T, J.$$

This method derives the equations of motion for a complicated mechanical system. It should be done using two variables.

U^* = co-energy directly related to the kinetic energy of the system components, these are capacitive elements electrically and mechanically mass. Kinetic energy of the entire system thus comprises the sum of the partial kinetic energies of each link with proper mass.

$$E_c = U^* = \frac{1}{2}mv^2$$

$$U^* = \frac{1}{2}mv^2_1 + \frac{1}{2}mv^2_2 \quad (9)$$

The expression (9) links the angular speed with the linear speed:

- v = linear speed link
- $w = \dot{\theta}$ = angular speed
- θ = angular displacement
- l = length of each link

$$v = wl \quad (10)$$

- (7) Replacing the expression (10) in (9) to obtain an expression in terms of the system parameters, they are:

$$m_n, l_n, \theta_n.$$

$$U^* = \frac{1}{2}m_1 (l_1\dot{\theta}_1)^2 + \frac{1}{2}m_2 \left((l_1\dot{\theta}_1)^2 + (l_2\dot{\theta}_2)^2 + 2l_1l_2\dot{\theta}_1\dot{\theta}_2 \cos \theta_2 \right) \quad (11)$$

$$\left. \begin{aligned} T &= \frac{1}{2}kx^2 \\ J &= \frac{1}{2}Bv^2 \end{aligned} \right\} = 0$$

$$L = U^* - T; \quad T = 0$$

$$L = U^*$$

Non dissipative and inductive elements or G and L which are zero and calculating Lagrangian depends only on the co-energy stored in capacitive effect devices such as each link of the robotic arm with two degrees of freedom because it has masses own.

$$L = \frac{1}{2}m_1 (l_1\dot{\theta}_1)^2 + \frac{1}{2}m_2 \left((l_1\dot{\theta}_1)^2 + (l_2\dot{\theta}_2)^2 + 2l_1l_2\dot{\theta}_1\dot{\theta}_2 \cos \theta_2 \right) \quad (12)$$

- (8) For each variational coordinate apply the Euler-Lagrange equation, the Lagrangian L is subjected to referral for each acting variable on the system.

$$\frac{d}{dt} \left[\frac{\partial L}{\partial \dot{\theta}_1} \right] - \frac{\partial L}{\partial \theta_1} + \frac{\partial J}{\partial \dot{\theta}_1} = \tau_1$$

$$\frac{d}{dt} \left[\frac{\partial L}{\partial \dot{\theta}_1} \right] = \frac{d}{dt} \left[\frac{\partial}{\partial \dot{\theta}_1} \left(\frac{1}{2}m_1 (l_1\dot{\theta}_1)^2 + \frac{1}{2}m_2 \left((l_1\dot{\theta}_1)^2 + (l_2\dot{\theta}_2)^2 + 2l_1l_2\dot{\theta}_1\dot{\theta}_2 \cos \theta_2 \right) \right) \right]$$

$$\frac{d}{dt} \left[\frac{\partial L}{\partial \dot{\theta}_1} \right] = [m_1 l_1^2 \ddot{\theta}_1 + m_2 l_1^2 \ddot{\theta}_1 + m_2 l_1 l_2 \ddot{\theta}_2 \cos \theta_2 - m_2 l_1 l_2 \dot{\theta}_2 \sin \theta_2] = \tau_1$$

$$g_1(\theta) = 0$$

$$g_2(\theta) = 0$$

$$(m_1 + m_2) l_1^2 \ddot{\theta}_1 + m_2 l_1 l_2 (\ddot{\theta}_2 \cos \theta_2 - \dot{\theta}_2 \sin \theta_2) = \tau_1 \quad (13)$$

For θ_2 :

$$\frac{d}{dt} \left[\frac{\partial L}{\partial \dot{\theta}_2} \right] - \frac{\partial L}{\partial \theta_2} + \frac{\partial J}{\partial \dot{\theta}_2} = \tau_2$$

$$\frac{d}{dt} \left[\frac{\partial L}{\partial \dot{\theta}_2} \right] = \frac{d}{dt} \left[\frac{1}{2} m_2 l_2^2 * 2 * \dot{\theta}_2 + \frac{1}{2} * m_2 * 2 * l_1 l_2 \dot{\theta}_1 \cos \theta_2 \right]$$

$$\frac{\partial L}{\partial \theta} = -\frac{1}{2} m_2 * 2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin \theta_2$$

$$m_2 l_2 \ddot{\theta}_2 + m_2 l_1 l_2 (\ddot{\theta}_1 \cos \theta_2 - \dot{\theta}_1 \sin \theta_2) + m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin \theta_2 = \tau_2 \quad (14)$$

Equations (13) and (14) are the differential equations that representing the system, and they are obtained by differentiating the Lagrangian with respect of each variational coordinate obtained from the sum of energies that act on the system.

- (9) State variables Model given the parameters of the system allows the nonlinear model of proposed system.

$H(\theta)$ = inertia matrix

$c(\theta, \dot{\theta})$ = matrix centrifugal and Coriolis accelerations

$g(\theta)$ = gravity matrix

τ = torque or twisting torque applied to each link

$$H(\theta) * \ddot{\theta} + c(\theta, \dot{\theta}) + g(\theta) = \tau$$

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \quad (15)$$

Equation(15) represents a simplified general dynamic model of a robot.

$$h_{11}(\theta) = (m_1 l_1^2 + m_2 l_1^2)$$

$$h_{12}(\theta) = m_2 l_1 l_2 \cos \theta_2$$

$$h_{21}(\theta) = m_2 l_1 l_2 \cos \theta_2$$

$$h_{22}(\theta) = m_2 l_2^2$$

$$c_{11}(\theta, \dot{\theta}) = 0$$

$$c_{12}(\theta, \dot{\theta}) = -m_2 l_1 l_2 \sin \theta_2$$

$$c_{21}(\theta, \dot{\theta}) = -m_2 l_1 l_2 \sin \theta_2 + m_2 l_1 l_2 \dot{\theta}_2 \sin \theta_2$$

$$c_{22}(\theta, \dot{\theta}) = m_2 l_1 l_2 \dot{\theta}_1 \sin \theta_2$$

3. DEVELOPMENT CASE STUDIES

The approach presented in the case studies covers areas of robotics, process and other areas of interest in the “Escuela Politécnica Nacional”, identification and robot modeling approach facilitates control solutions also allows testing in this model before implemented by simulation, generating research and resource depletion when placing controllers, actuators and other already known since the system behavior is available.

3.1 ROBOTINO ®

To obtain the mathematical model of the omnidirectional mobile robot Robotino is necessary to consider the three degrees of mobility it possesses due to its three wheels. Omnidirectional wheels, in addition to the basic movements back and forth, allowed complicated movements: movements in all directions, diagonally, laterally and even 360°.

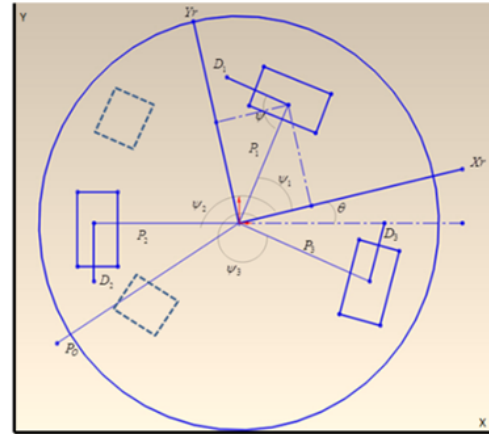


Figure 6: Robotino Geometry for the kinematic model.

Figure 6 shows a top view of the configuration of the mobile robot, this identifies the mobile framework $\{X_r, Y_r\}$ situated in the center of the vehicle. The fixed reference axis is the XY , which travel speeds Vector:

$$v_w = [v_1 \ v_2 \ v_3]^T \quad (16)$$

Unit vector:

$$D_1 \cdot X_r = |D_1| \cdot |X_1| \cos(\psi_1 + \psi) \quad (17)$$

$$D_1 \cdot X_r = \cos(\psi_1 + \psi)$$

The speed of each wheel and the speed of the robot is expressed by rotation of a unit vector in the direction X_r .

$$v_i = \begin{bmatrix} \cos(\varphi_i + \psi) & \sin(\varphi_i + \psi) \\ \sin(\varphi_i + \psi) & -\cos(\varphi_i + \psi) \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos(\varphi_i + \psi) \\ \sin(\varphi_i + \psi) \end{bmatrix} \quad (18)$$

Equation (18) indicates the movement of the wheels and the robot without considering the rotation angle which arises in equation (19) as the rotation part in the cosine and the tangential rotation Ω , according to movement of omnidirectional wheels $\psi = 90^\circ = \pi/2$. [2]

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \frac{1}{r} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \frac{1}{r} \begin{bmatrix} -\sin(\theta - \varphi_1) & \cos(\theta - \varphi_1) & R \\ -\sin(\theta - \varphi_2) & \cos(\theta - \varphi_2) & R \\ -\sin(\theta - \varphi_3) & \cos(\theta - \varphi_3) & R \end{bmatrix} \cdot \begin{bmatrix} \dot{x}_w \\ \dot{y}_w \\ \Omega \end{bmatrix} \quad (19)$$

The dynamic model of the mobile robot is obtained from the Euler-Lagrange method. For which it is considered that the center of mass G is at the origin of the reference axes mobile.

It follows the dynamic model of the system in the form:

$$D\ddot{Q} + C(\dot{Q})\dot{Q} = B\tau \quad (20)$$

$$D = \begin{bmatrix} \frac{3Ir}{2r^2} + M_R & 0 & 0 \\ 0 & \frac{3Ir}{2r^2} + M_R & 0 \\ 0 & 0 & \frac{3Ir}{2r^2} + M_R \end{bmatrix} \quad C(\dot{Q}) = \frac{3Ir}{2r^2} \begin{bmatrix} 0 & -\dot{\varphi} & 0 \\ \dot{\varphi} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\beta = \frac{1}{r} \begin{bmatrix} -\sin(\theta + \varphi) & -\sin(\theta + \varphi) & \cos\varphi \\ -\cos(\theta + \varphi) & -\cos(\theta + \varphi) & \sin\varphi \\ R & R & R \end{bmatrix}$$

3.2 Helicopter Two Degrees of Freedom

A helicopter flies by a plane same principles, but in the case of helicopters lift is achieved by rotating blades. These make the lift possible. Its shape produces a force when the air passes through them. The rotor blades are profiles designed specifically for the flight characteristics. [3]

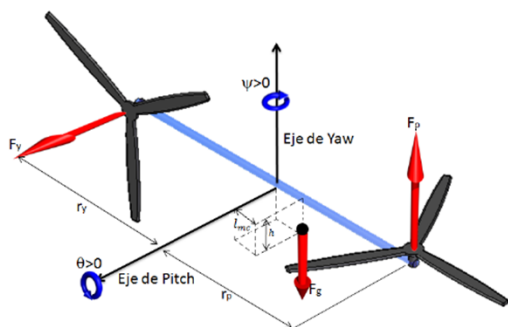


Figure 7: Helicopter Dynamics [4].

The chopper rotates over an angle ψ Z and Y with respect to an angle θ , so it considers the position of the center of mass with respect to a fixed coordinate system [5].

We use homogeneous transformation matrixes

$$Rot_{z_0, \psi} \text{ y } Rot_{y_1, \theta}$$

$$T^2 = Rot_{z_0, \psi} Rot_{y_1, \theta} \begin{bmatrix} 1 & 0 & 0 & l_{mc} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -h \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (21)$$

Is linearized around $\bar{\theta} = 0$

$$A(0) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{M_{heli}gh}{[J_{eqp} + M_{heli} + (l_{mc}^2 + h^2)]} & 0 & -\frac{B_p}{[J_{eqp} + M_{heli} + (l_{mc}^2 + h^2)]} & 0 \\ 0 & 0 & 0 & -\frac{B_y}{[J_{eqy} + M_{heli} + l_{mc}^2]} \end{bmatrix}$$

$$B(0) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{K_{pp}}{[J_{eqp} + M_{heli} (l_{mc}^2 + h^2)]} & \frac{K_{py}}{[J_{eqp} + M_{heli} (l_{mc}^2 + h^2)]} \\ \frac{K_{yp}}{[J_{eqy} + M_{heli} l_{mc}^2]} & \frac{K_{yy}}{[J_{eqy} + M_{heli} l_{mc}^2]} \end{bmatrix} \quad (22)$$

The values obtained for the state variables depends on the value we assign to $\bar{\theta}$.

3.3 Identification in Time Tranche Pipeline

The section of pipeline that experimental data is discussed was taken from SOTE (Sistema del Oleoducto Trans Ecuatoriano) and the test was done in normal operating conditions, a change is made passage of small amplitude set point as an unscheduled stop produce consequences of time and money [6].

In the time domain response of a non-oscillatory system has the form:

$$C(t) = A + k_1 e^{-t/\tau_1} + k_2 e^{-t/\tau_2} + k_3 e^{-t/\tau_3} + \dots + k_n e^{-t/\tau_n} \quad (23)$$

For a proper identification system requires multiple iterations approaching a line of a first order type.

Where:

- A = variation amplitude of the output.
- τ_i = system time constants.
- n = order system.
- k_i = constant of proportionality.

Table 2. Variation of discharge pressure function of time.

TIME IN	Δ DISCHARGE
0	0
2	1
6	16
11	35
12	42
14	49
15	53
17	62
18	65
22	79
24	85
28	92
31	96
33	99
36	102
39	105
43	107
46	108
49	110
52	111
56	112

If any dominant time constant is so large times the terms of equation (23) having small time constants tend to zero, while the end of the large time constant is still different from zero, so that approaches:

$$C(t) \approx A + k_1 e^{-t/\tau_1} + 0 + 0 + \dots + 0 \quad (24)$$

$$C(t) = A + k_1 e^{-t/\tau_1} \quad (25)$$

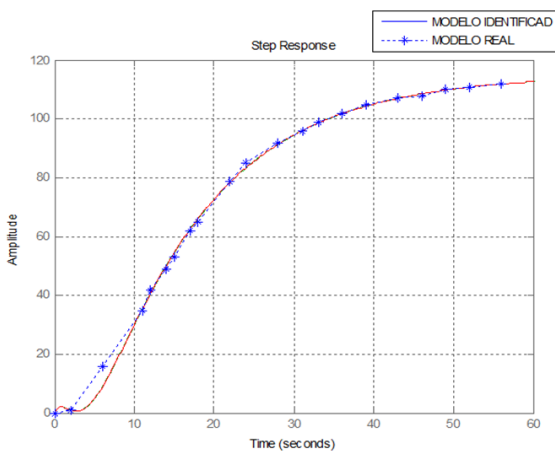


Figure 8: Step function response, identifying the system.

According to the actual model graphic representation of the variation of the discharge pressure versus time, when excited closed loop system a step function, and the model identified according to the data transfer function allows to validate de transfer function models after a few iterations.

4. SIMULATION SOFTWARE RESULTS

The developed software enables configurable data entry for each model, one that can provide a stable response in open loop systems without incurring the deduction of mathematical models and long hours of programming. This software was developed in order to have an understanding of the modeling and identification [7] [8].

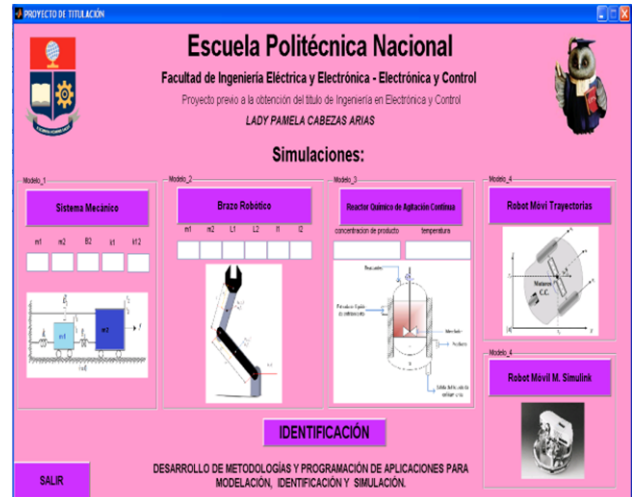


Figure 9: Software Simulation and Identification.

To use the software properly you must click on the button the system to which access is required, for example Mechanical, and place the required value for each field, the characteristics vary according to the parameters are entered. All fields are required otherwise the simulation display an error message.

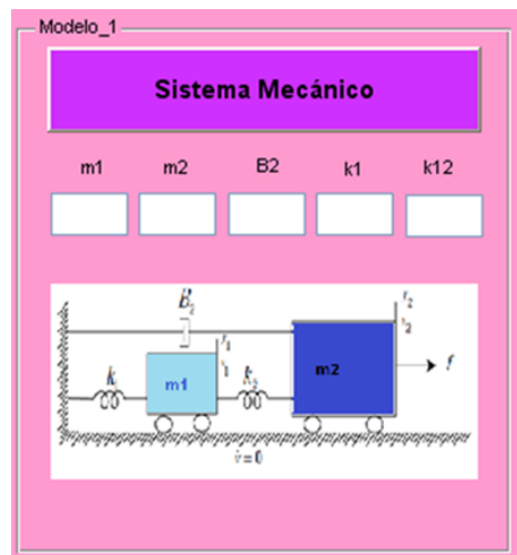


Figure 10: Model Simulation Software 1 [11].

All fields are mandatory to fill in accessing output parameters, otherwise the system will not allow the progress of the simulation and generates a series of errors reminding the user that the fields are mandatory.

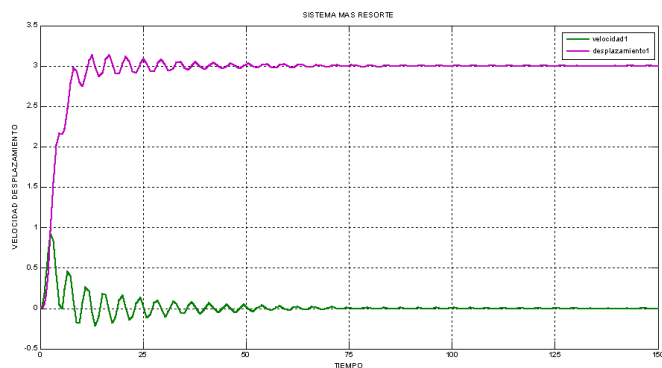


Figure 11: Example of speed and displacement [9].

Figure 11 shows the displacement and speed response to which is subject to two springs one to the right and to the left so that the displacement is oscillatory due to the motion presented by the mass in the system, the maximum has mass1 speed of 0.8 m / s and stabilization time is 75 seconds [10].

5. CONCLUSIONS

The most appropriate method in modeling is the variational system, it allows state variables from nonlinear systems as on simplified model of an manipulator.

A system can be stable despite being in open loop and presenting a high settling time as in de two masses system. For the kinematic model Robotinos® in inverse kinematics is used and it is necessary to establish a referral system that identifies each wheel of the robot because without it the angles between the wheels cannot be analyzed by his

unit vectors for tangential speed define its homogeneous matrix. The method of Newton-Euler modeling is useful in mechanical systems considering the forces and torques acting on the system, but has restrictions between the relations of forces present, so that math becomes complex.

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